Let A_0 be an arbitrary bounded linear operator in a Hilbert space, $A = A_0 + B$, B an operator from the Schatten - von Neumann class S_p, $p > 0$. By Weyl's theorem, for the essential spectra $\sigma_{ess}(A) = \sigma_{ess}(A_0)$, and under some additional assumptions on A_0

$$\sigma(A) = \sigma_{ess}(A) \bigcup \sigma_d(A)$$

holds, where $\sigma_d(A) = \{\lambda_j\}$ is the discrete spectrum of A, which is at most countable set of isolated eigenvalues of a finite algebraic multiplicy with all accumulation points on $\sigma_{ess}(A)$.

The problem is to find quantitative estimates for the rate of condensation, in other words, inequalities of the type

$$\sum_j \operatorname{dist}^q(\lambda_j, \sigma(A_0)) < C\|B\|^p_{S_p}.$$

We get such kind of results under the following assumptions upon A_0:

(1) $\sigma(A_0) = \sigma_{ess}(A_0)$, the compact set $\sigma(A_0)$ does not split the plane (its complement is connected), and is r-convex. The latter is a pure geometric characteristic of a compact set, that generalizes the usual notion of convexity. For example, each compact set on a line or a circle is r-convex for some $r > 0$.

(2) The resolvent $R(\lambda, A_0)$ has a polynomial growth when approaching the spectrum

$$\|R(\lambda, A_0)\| \leq \frac{C}{\operatorname{dist}^s(\lambda, \sigma(A_0))}, \quad s > 0, \quad \lambda \notin \sigma(A_0).$$

In particular, the latter holds for normal (subnormal, spectral in the sense of Dunford etc.) operators.

The above setting includes the problem about the spectrum of operators with the imaginary component from S_p.

The existence of the perturbation determinant enables one to apply here the methods of potential theory. More precisely, we obtain the Blaschke-type conditions on the Riesz measure of subharmonic functions in unbounded domains with r-convex compact complement, that grow polynomially near the boundary. The last result has an independent interest in the function theory.