The University Of Sheffield.

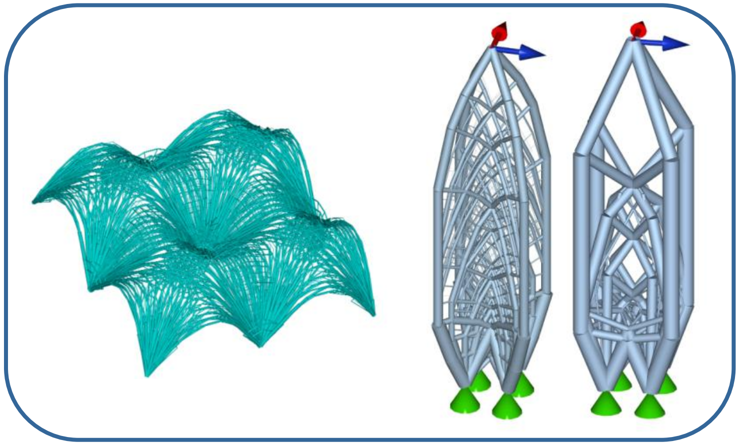
SIAM Student Chapter Day, Cardiff, January 21st 2013

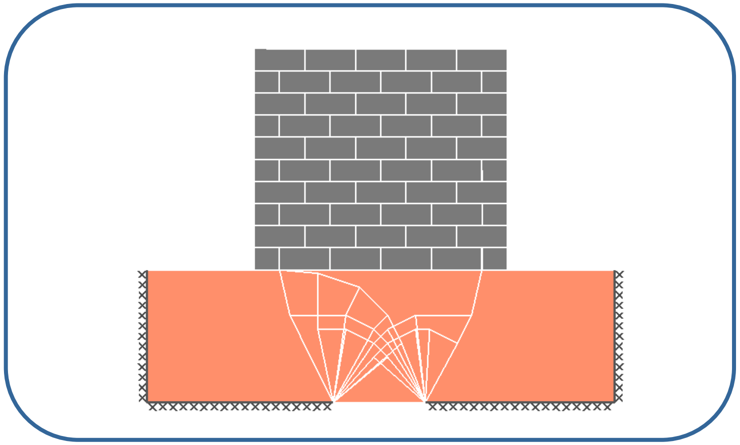
Application of Layout Optimization to Engineering Analysis and Design Problems

Matthew Gilbert
Dept. Civil & Structural Engineering
University of Sheffield, UK

Contents

- Industry context
- What is 'layout optimization'?
- Applications:
  - I. Design optimization
  - II. Collapse analysis
- Current work
- Conclusions





Industry context

The University Of Sheffield.

Industry context

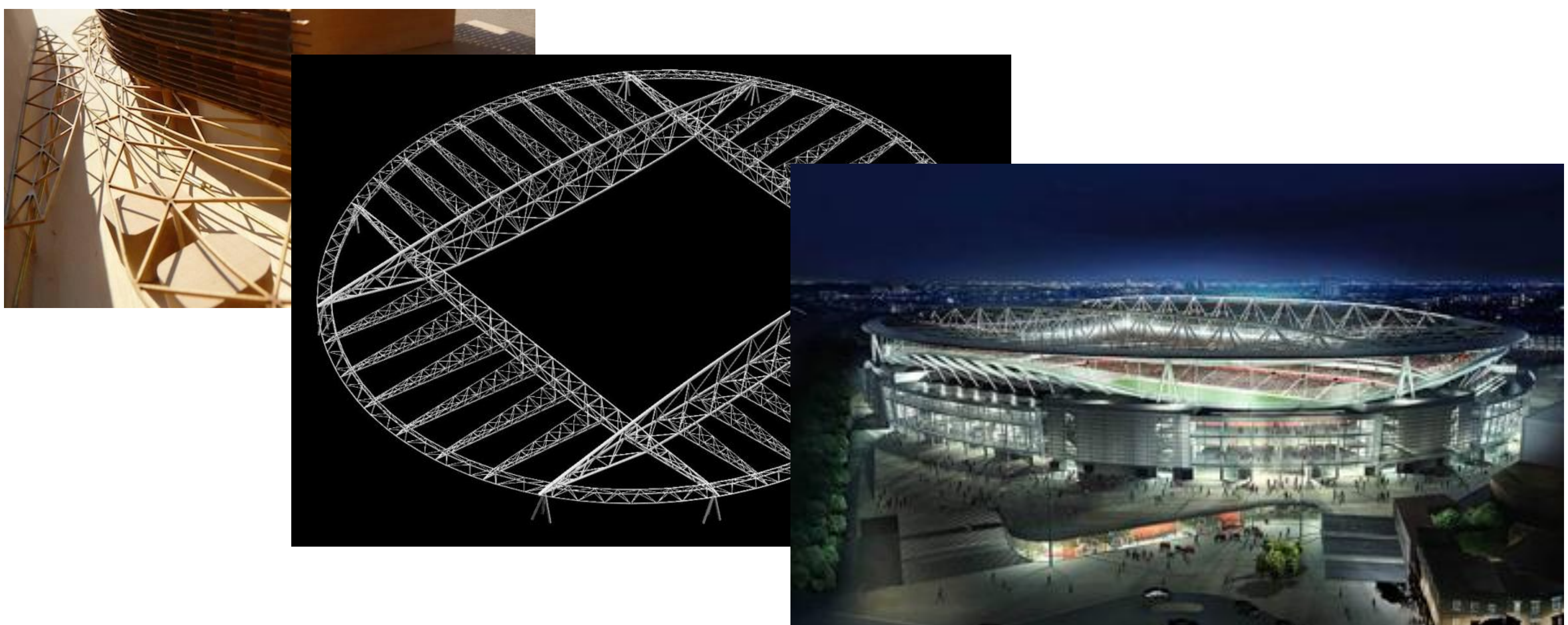
- **The UK construction industry is worth >£100bn per annum (almost 10% of GDP):**
  - **Half spent on new construction, mostly ‘one off’ projects**
  - **Half spent on existing infrastructure, often dealing with ‘one off’ structures and ‘difficult’ materials (e.g. masonry)**

Typically limited time to get highly refined solutions, but need rapid & effective means of getting close…

The University Of Sheffield.

Existing tools for design

- Laborious manual process, e.g. for layout of elements forming stadium roof:

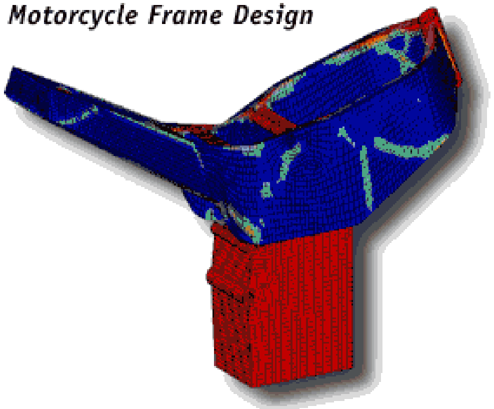


The University Of Sheffield.

Existing tools for design (cont.)

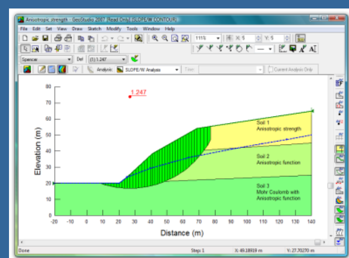
- Automated methods very rarely used by structural engineers for roofs etc.
- Continuum-based methods (e.g. see below) unlikely to be suitable

Motorcycle Frame Design



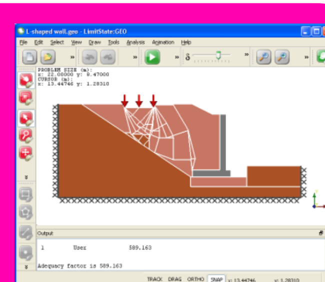
Existing tools for collapse analysis

‘Traditional’: based on hand analysis solutions etc.

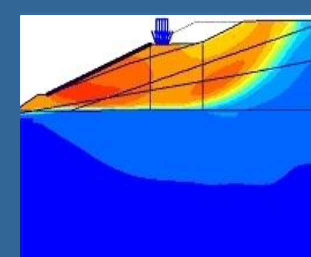


(potentially embedded in simple programs / spreadsheets etc.)

‘Mainstream’: based on computational limit analysis?



‘Advanced’: based on non-linear finite elements etc.



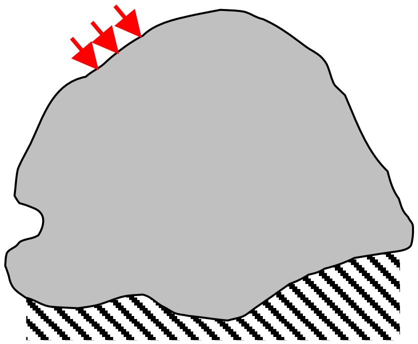
More:

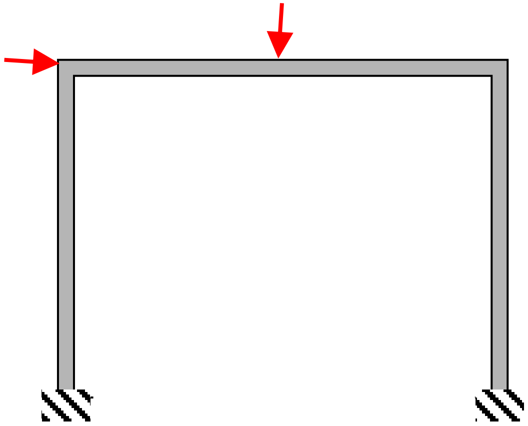
- complex
- time consuming
- input parameters
- expertise required
- accurate [potentially at least!]

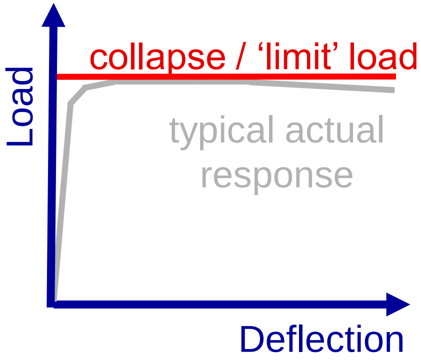
The University Of Sheffield.

What is 'limit analysis'?

- A method of estimating the maximum load sustainable by a body or structure



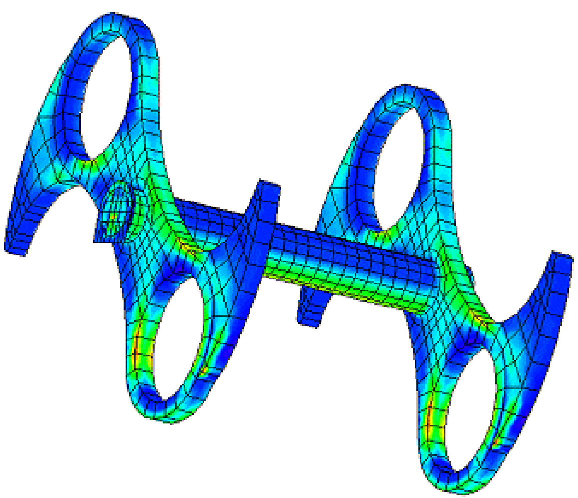


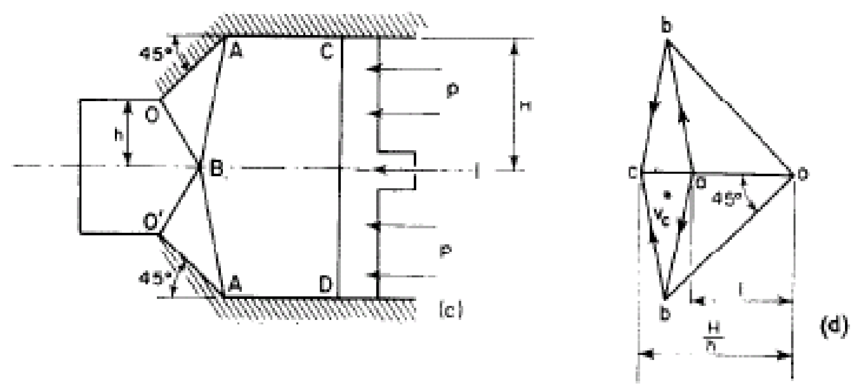


The University Of Sheffield.

Elastic vs. plastic (limit) analysis

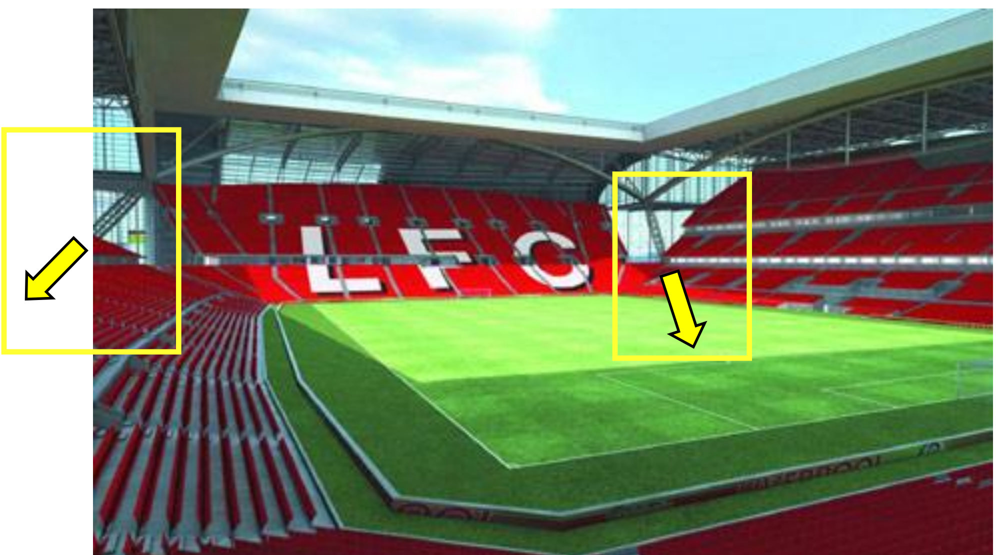
- Elastic analysis methods: finite elements have made analysis straightforward
- Plastic (limit) analysis methods: tools are much less well developed...

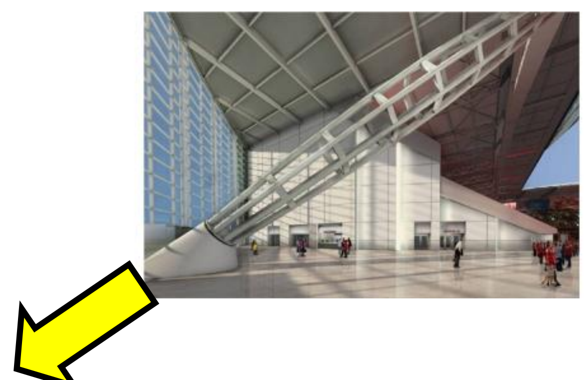




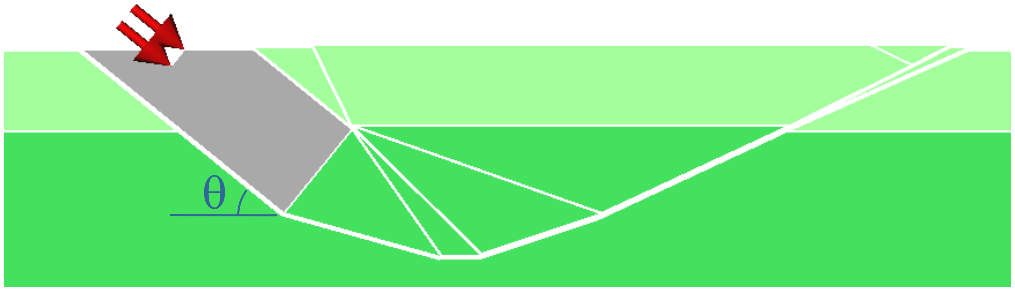
Example: stadium foundations

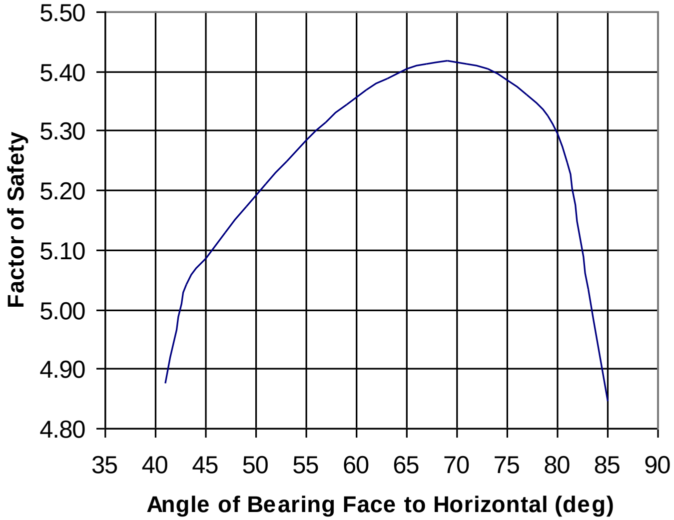
- Time-consuming **analysis** process to verify safety of proposed foundations





Example: stadium foundations [2]



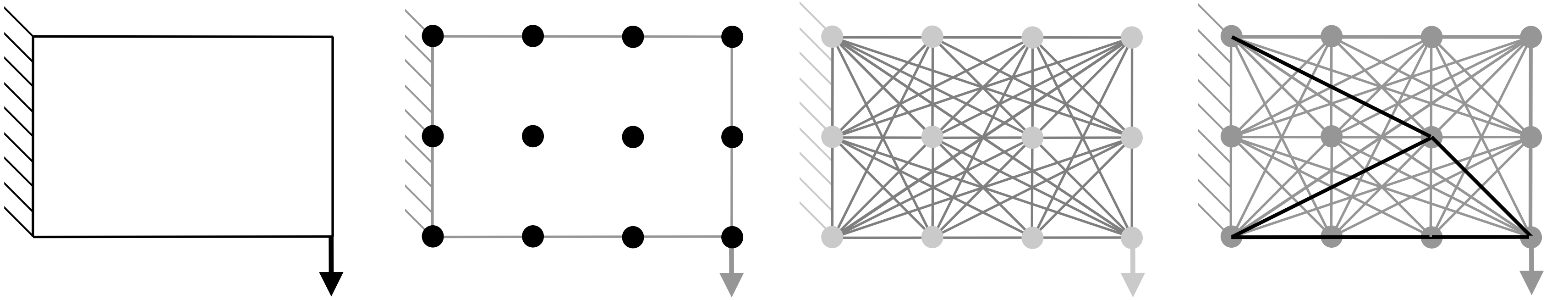


(analysis undertaken on LFC stadium foundations by Laing O'Rourke)

What is ‘layout optimization’?

What is ‘layout optimization’?

- Technique originally devised in the 1960s to find topology of minimum weight trusses:



(after Dorn, Gomory, and Greenberg, 1964)

Example I: design optimization (of trusses)

Primal Linear Programming formulation: trusses

$$\min \ V = \mathbf{q}^{\mathrm{T}}\mathbf{c}$$

minimise volume

subject to:

applied nodal forces

$$\mathbf{B}\mathbf{q} = \mathbf{f}$$

internal bar forces

where:

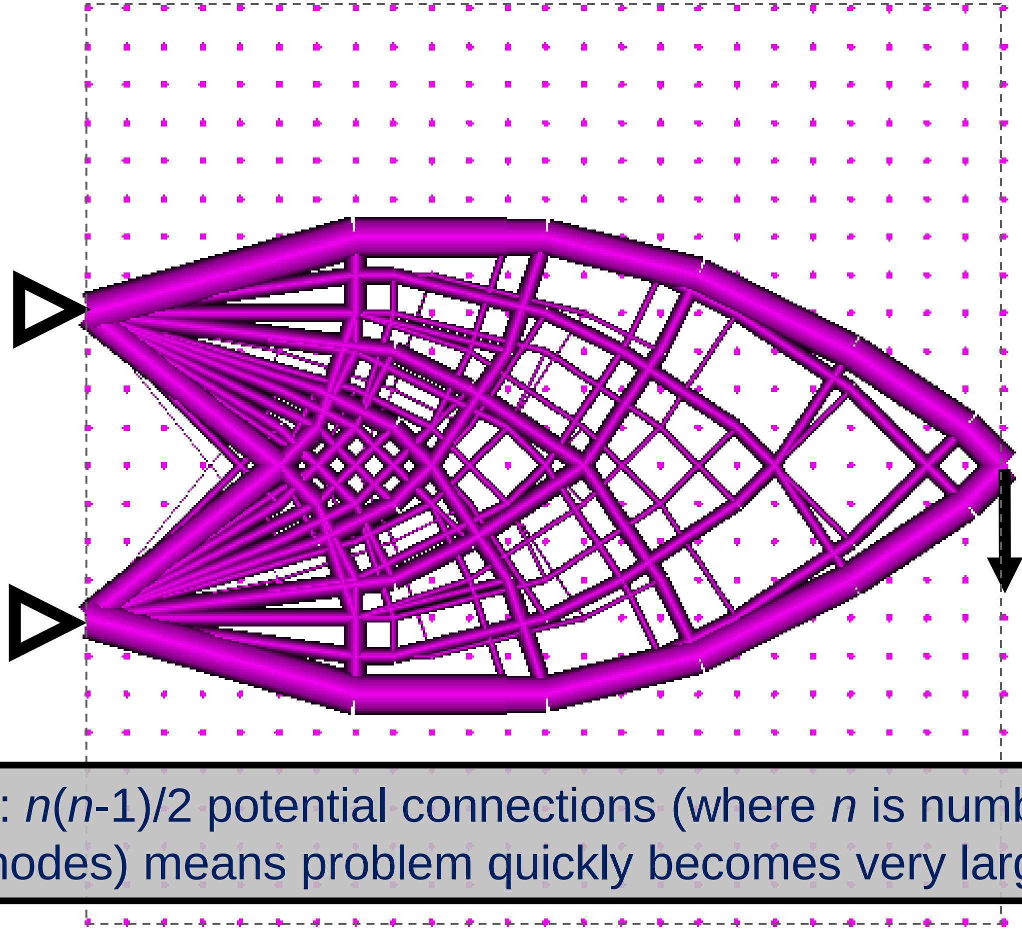
$$q_i^+, \ q_i^- \geq 0$$

$$\mathbf{c}^{\mathrm{T}} = \{l_1/\sigma_1^+, -l_1/\sigma_1^-, l_2/\sigma_2^+, -l_2/\sigma_2^-, \ldots -l_m/\sigma_m^-\}$$

B = equilibrium matrix

The University Of Sheffield.

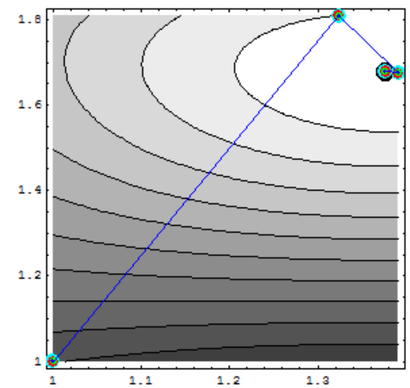
Example: Hemp cantilever



But: *n*(*n*-1)/2 potential connections (where *n* is number of nodes) means problem quickly becomes very large!

The University Of Sheffield.

On the use of 'adaptivity'...



- Mathematical programming problems can quickly become computationally expensive
- Sheffield approach:
  1. Formulate simple reduced problem, ideally solvable using LP (e.g. MOSEK), then adaptively refine
  2. General software framework to take advantage of problem similarities

The University Of Sheffield.

Dual Linear Programming formulation: trusses

$$\max \; W = \mathbf{f}^{\mathrm{T}}\mathbf{u}$$

maximise work done by external applied loads

subject to:

$$-\frac{l_i}{\sigma_i^-} \leq \mathbf{B}^{\mathrm{T}}\mathbf{u} \leq \frac{l_i}{\sigma_i^+}$$

where: $\mathbf{u}^{\mathrm{T}} = \{u_1^x, u_1^y, u_2^x, u_2^y, \ldots u_n^y\}$

nodal displacements (LP variables)

equivalent to the Michell-Hemp optimality criteria: $-\frac{1}{\sigma_i^-} \leq \varepsilon_i \leq \frac{1}{\sigma_i^+}$

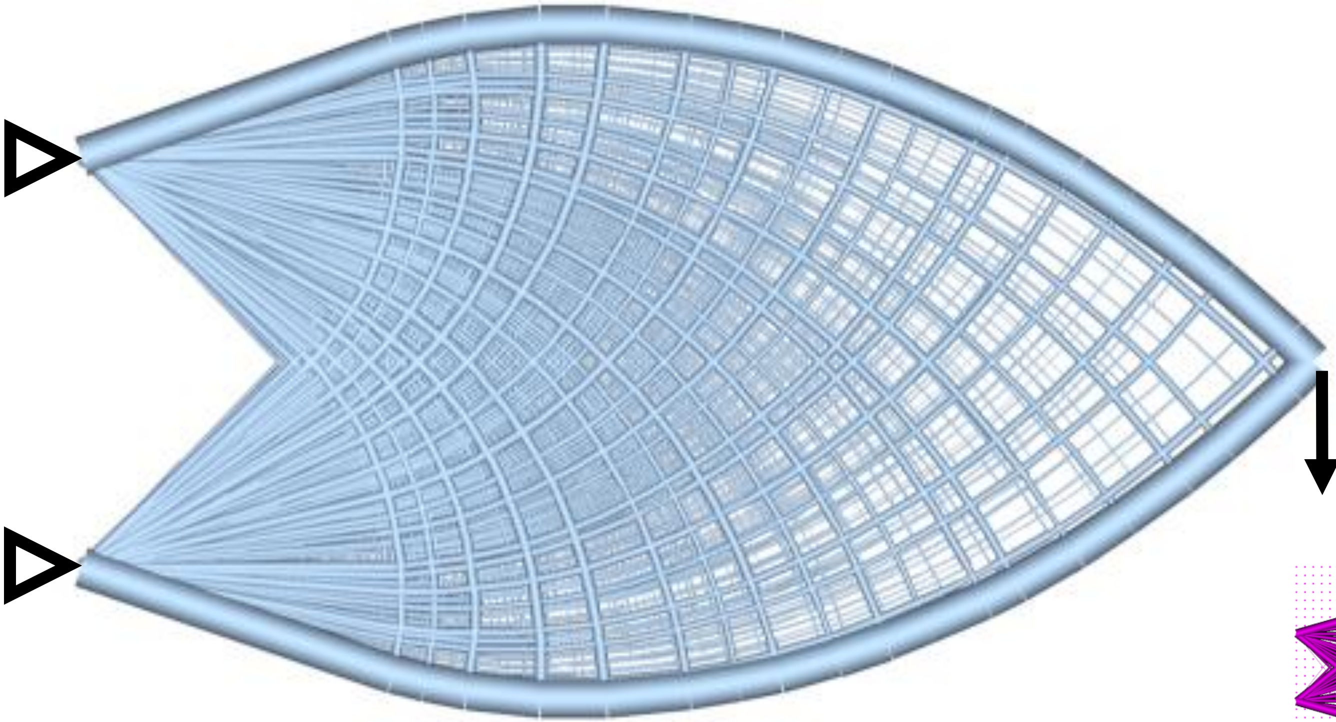
Adaptive solution…

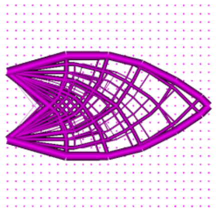
- Procedure (Gilbert & Tyas, Eng. Comp. 2003):
  1. Connect nodes to adjacent nodes only (say)
     ⇒ obtain an initial sub-problem
  2. Solve (using LP)
  3. Identify which potential members most violate the Michell-Hemp Optimality Criterion:

$$-\frac{1}{\sigma_i^-} \leq \varepsilon_i \leq \frac{1}{\sigma_i^+}$$

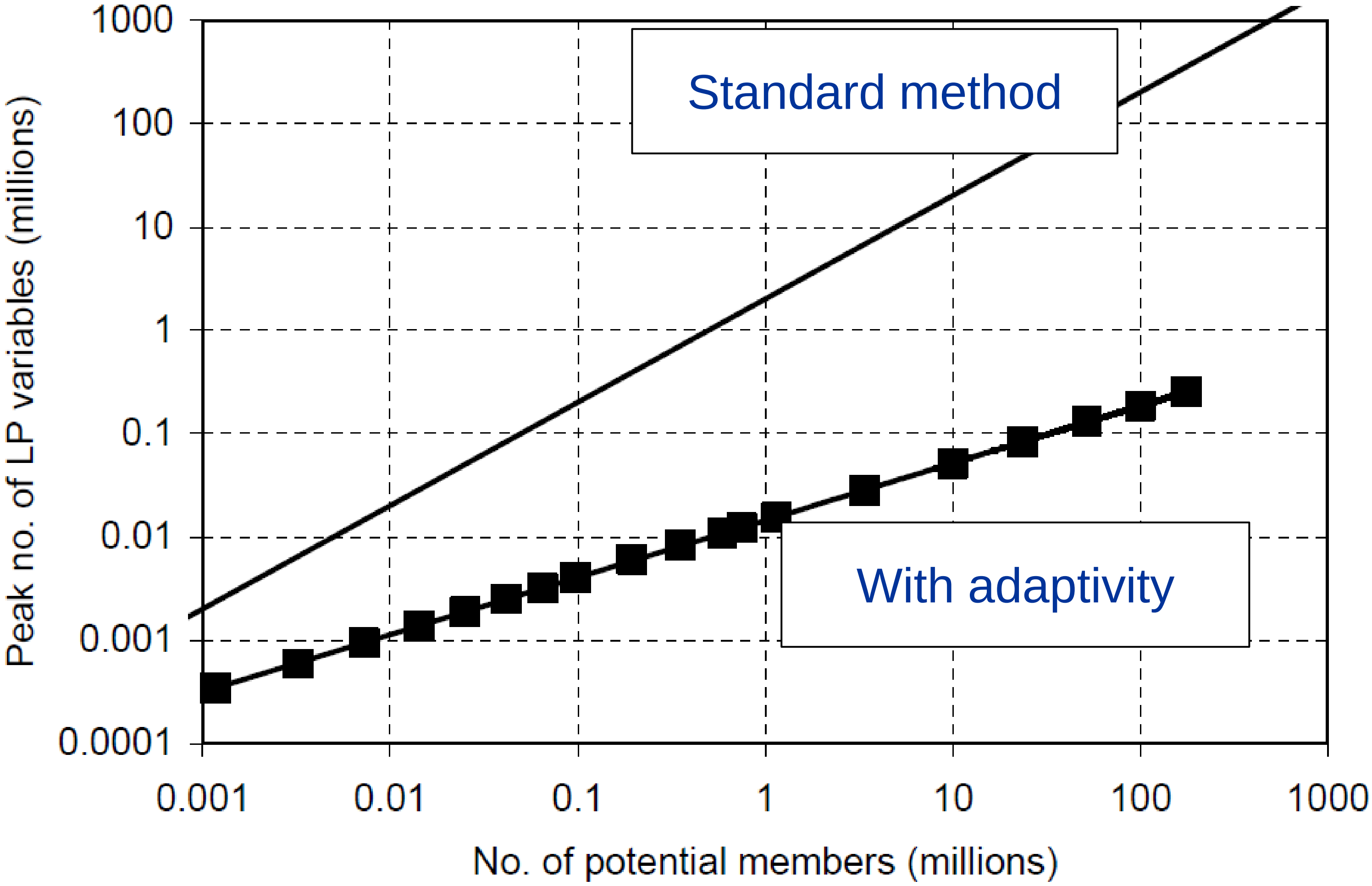
  4. Add members
     ⇒ obtain a revised sub-problem
  5. Repeat from 2.

Example: Hemp cantilever

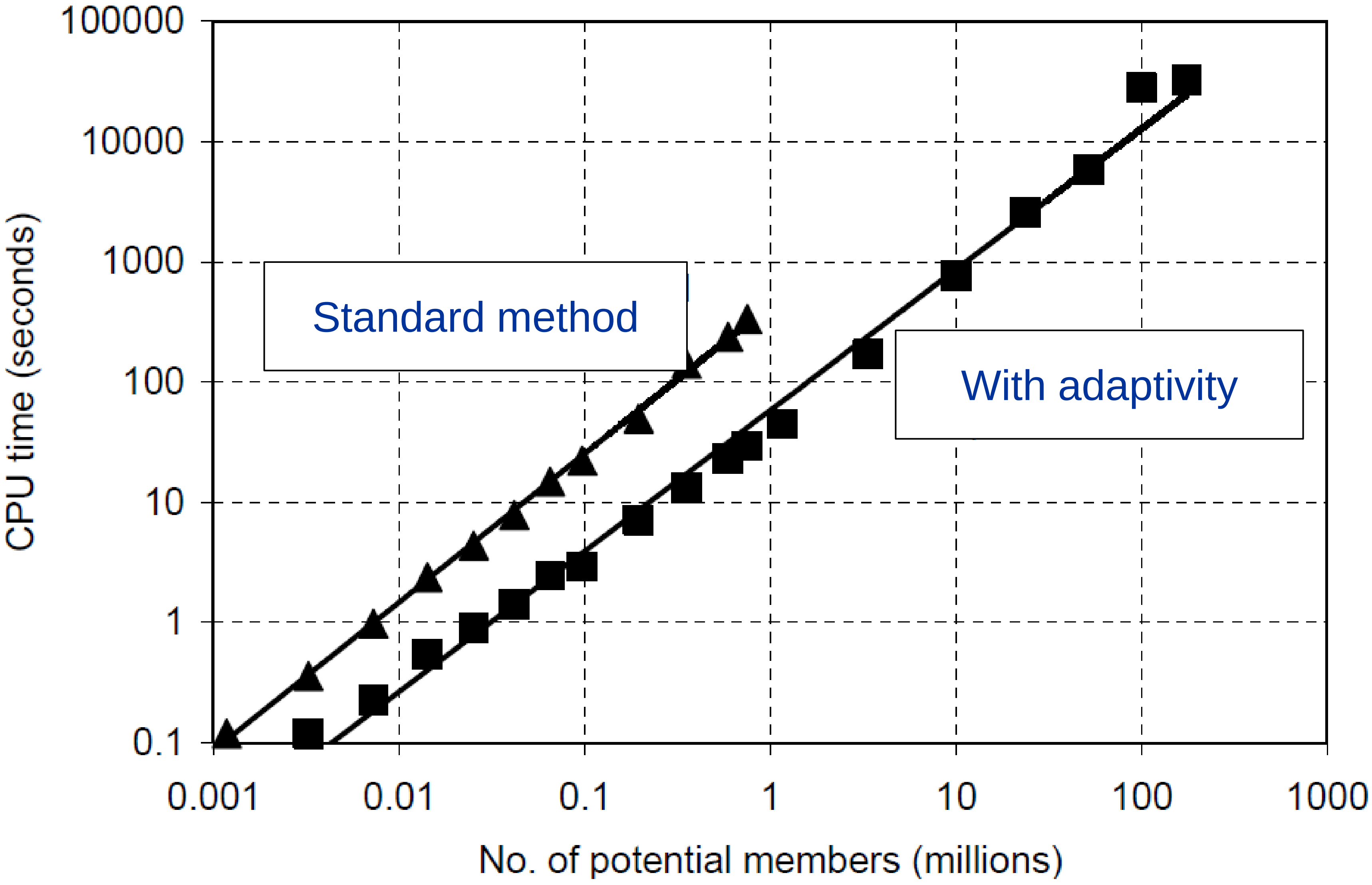




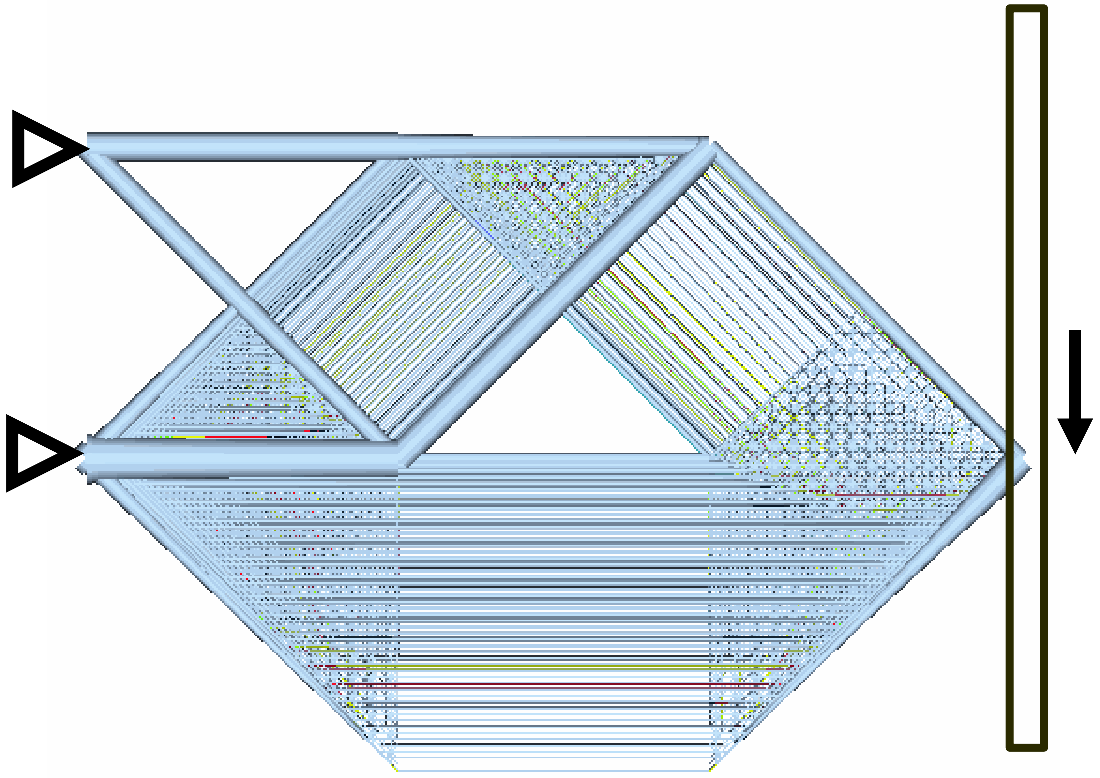
The University Of Sheffield.



The University Of Sheffield.



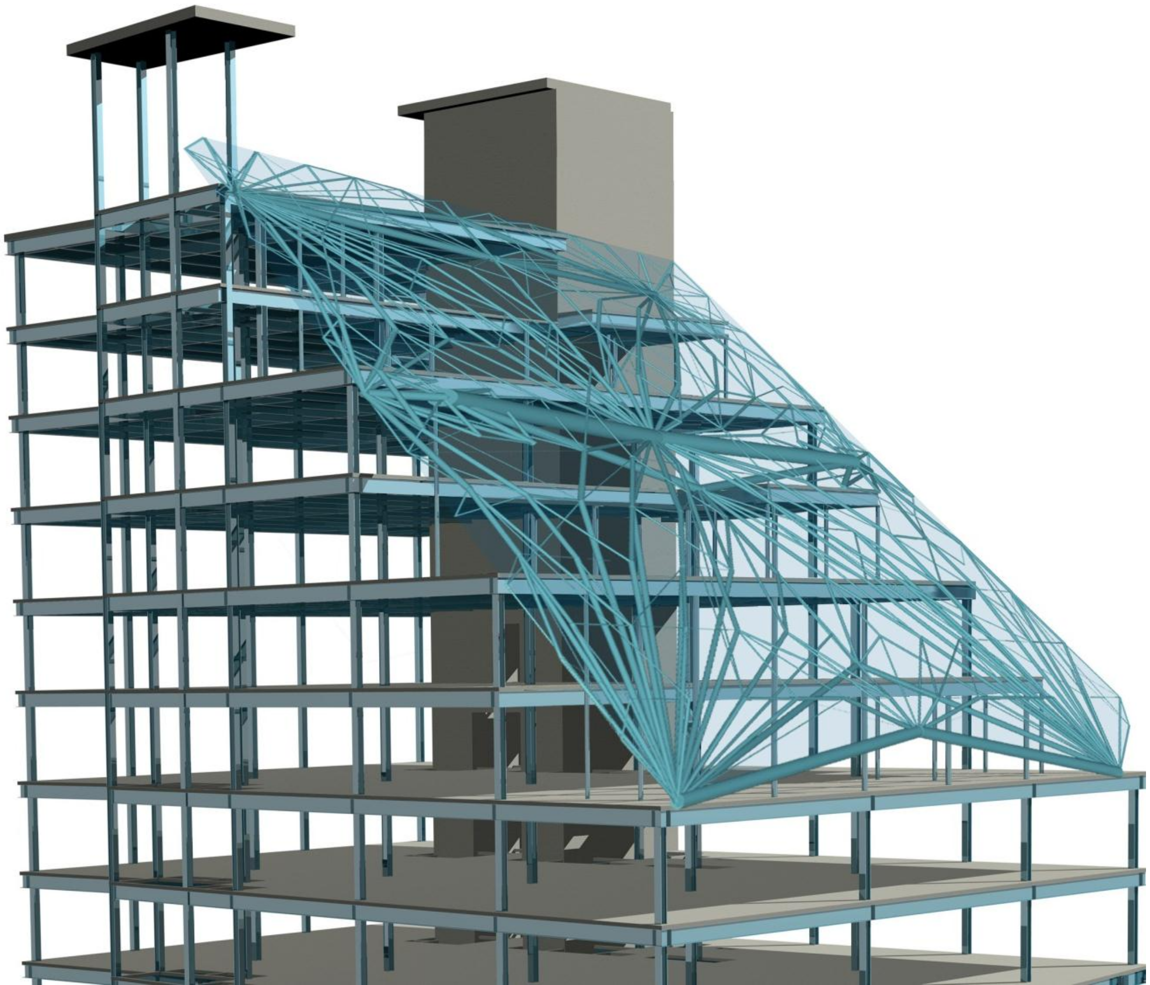
Progress of adaptive solution



Proof of Optimality

- *If, after an iteration, there are no potential members which violate the Michell-Hemp optimality criterion, by definition the current optimal solution must also be the optimal solution for the fully connected ground structure.*

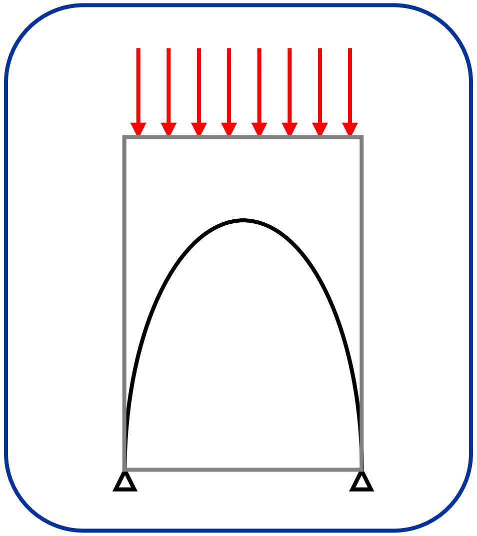
Extension: roof loading

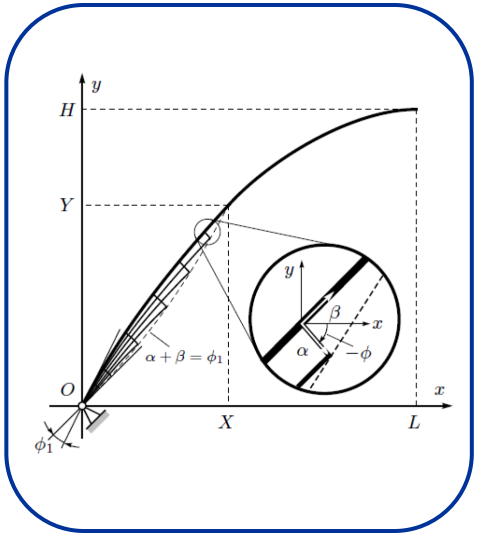


The University Of Sheffield.

Extension: roof loading

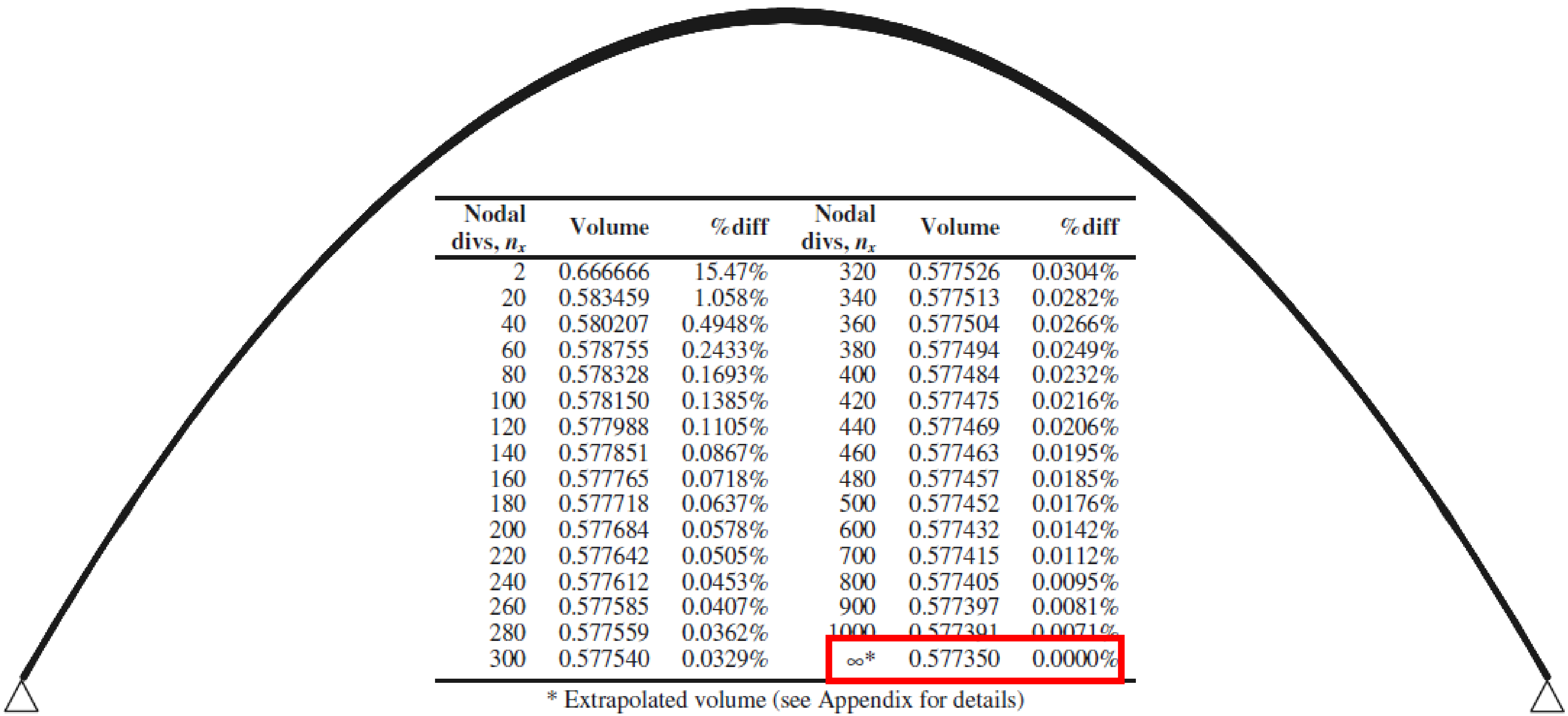
- Issue over where loads applied
- But problems validating 'transmissible' load model...
- Led to interesting academic diversion, with surprising outcome...
- ...overturned centuries old belief that parabolic form most optimal to carry a uniform load
  (Darwich, Gilbert & Tyas, SMO 2010; Tyas, Pichugin & Gilbert, Proc. R. Soc. A. 2011)





The University Of Sheffield.

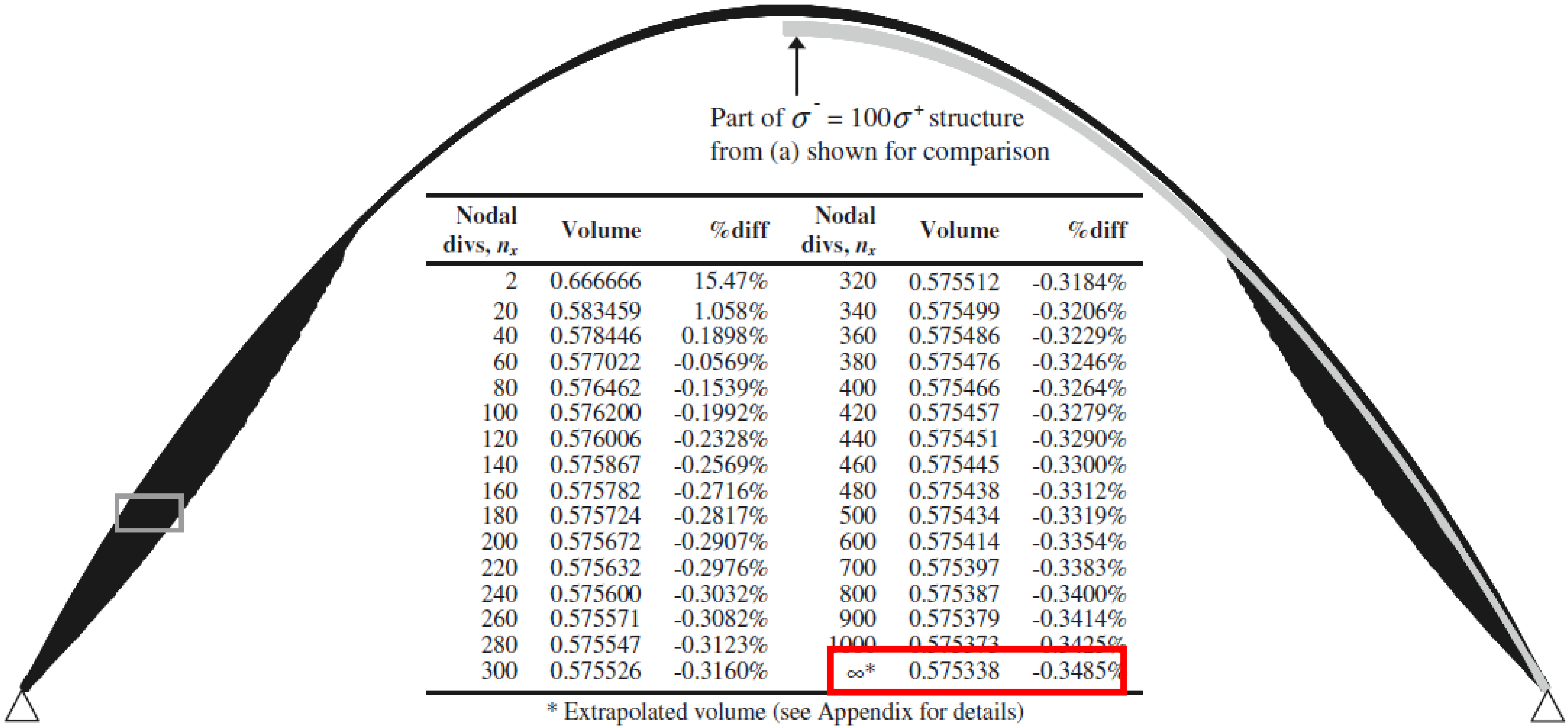
Numerical results, $\sigma^- = 100\sigma^+$:



Nodal divs, n_x	Volume	%diff	Nodal divs, n_x	Volume	%diff
2	0.666666	15.47%	320	0.577526	0.0304%
20	0.583459	1.058%	340	0.577513	0.0282%
40	0.580207	0.4948%	360	0.577504	0.0266%
60	0.578755	0.2433%	380	0.577494	0.0249%
80	0.578328	0.1693%	400	0.577484	0.0232%
100	0.578150	0.1385%	420	0.577475	0.0216%
120	0.577988	0.1105%	440	0.577469	0.0206%
140	0.577851	0.0867%	460	0.577463	0.0195%
160	0.577765	0.0718%	480	0.577457	0.0185%
180	0.577718	0.0637%	500	0.577452	0.0176%
200	0.577684	0.0578%	600	0.577432	0.0142%
220	0.577642	0.0505%	700	0.577415	0.0112%
240	0.577612	0.0453%	800	0.577405	0.0095%
260	0.577585	0.0407%	900	0.577397	0.0081%
280	0.577559	0.0362%	1000	0.577391	0.0071%
300	0.577540	0.0329%	∞*	0.577350	0.0000%

* Extrapolated volume (see Appendix for details)

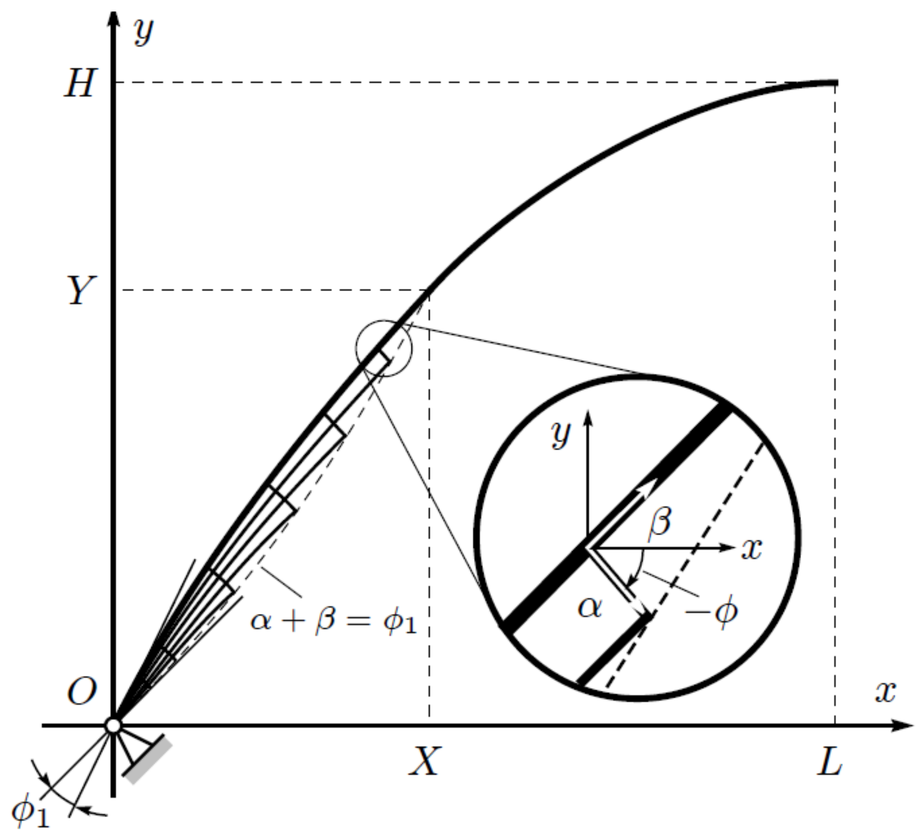
Numerical results, $\sigma^- = \sigma^+$:



Nodal divs, n_x	Volume	%diff	Nodal divs, n_x	Volume	%diff
2	0.666666	15.47%	320	0.575512	-0.3184%
20	0.583459	1.058%	340	0.575499	-0.3206%
40	0.578446	0.1898%	360	0.575486	-0.3229%
60	0.577022	-0.0569%	380	0.575476	-0.3246%
80	0.576462	-0.1539%	400	0.575466	-0.3264%
100	0.576200	-0.1992%	420	0.575457	-0.3279%
120	0.576006	-0.2328%	440	0.575451	-0.3290%
140	0.575867	-0.2569%	460	0.575445	-0.3300%
160	0.575782	-0.2716%	480	0.575438	-0.3312%
180	0.575724	-0.2817%	500	0.575434	-0.3319%
200	0.575672	-0.2907%	600	0.575414	-0.3354%
220	0.575632	-0.2976%	700	0.575397	-0.3383%
240	0.575600	-0.3032%	800	0.575387	-0.3400%
260	0.575571	-0.3082%	900	0.575379	-0.3414%
280	0.575547	-0.3123%	1000	0.575373	-0.3425%
300	0.575526	-0.3160%	∞*	0.575338	-0.3485%

* Extrapolated volume (see Appendix for details)

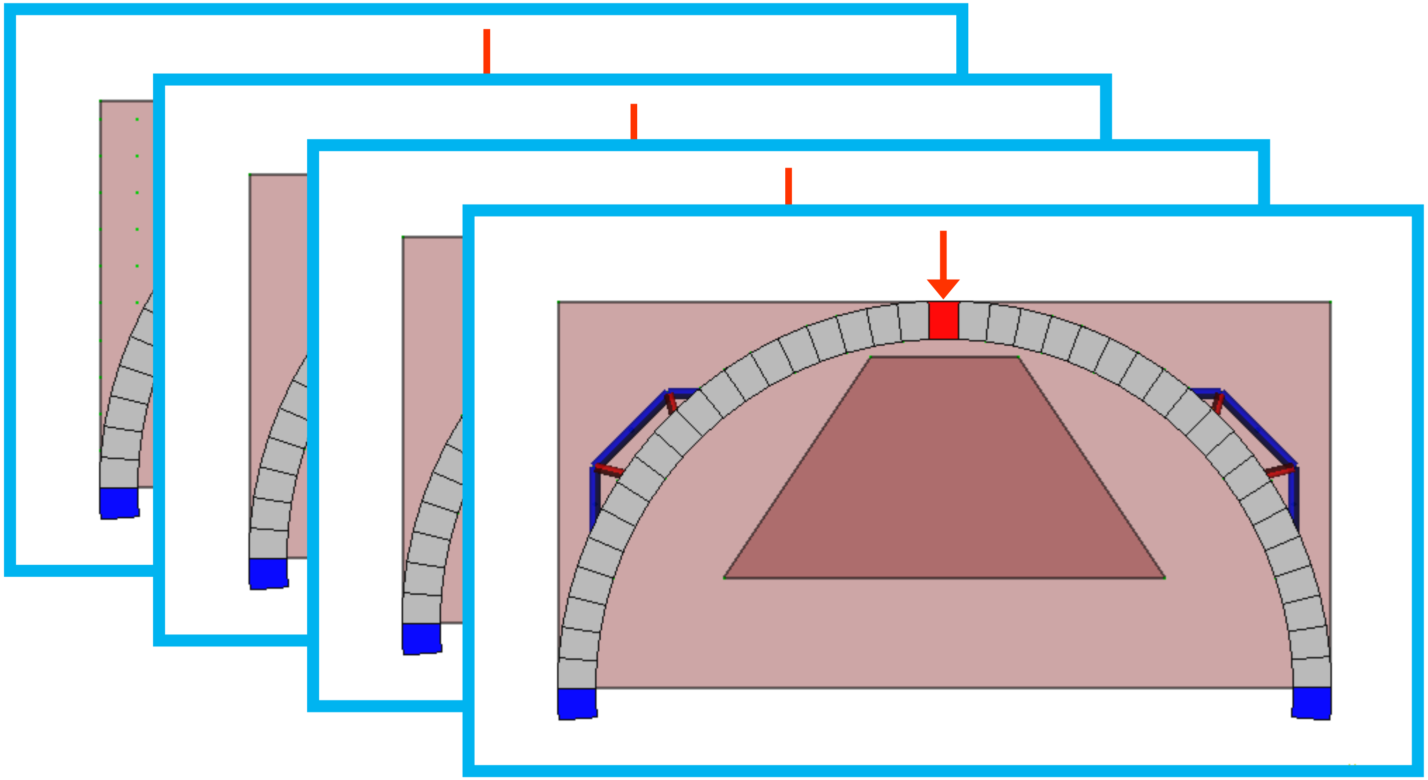
Analytical solution
(0.34<u>9</u>5% lighter than parabolic arch)



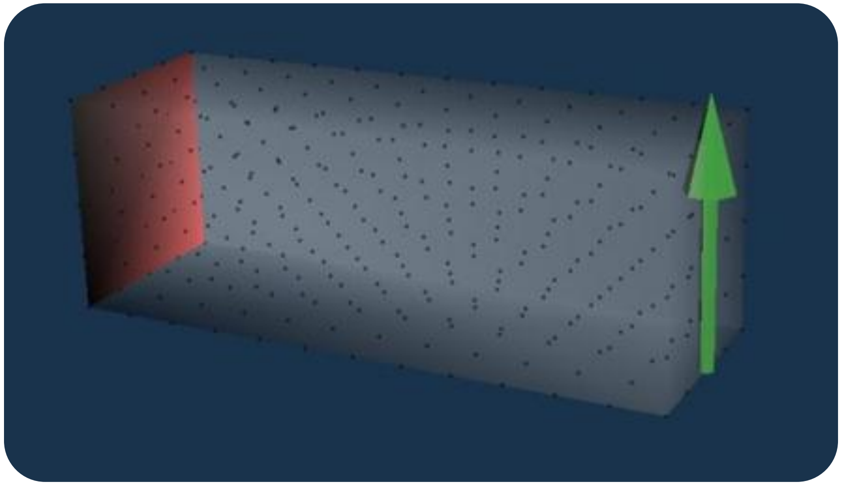
(Tyas, Pichugin & Gilbert, Proc. R. Soc. A. 2011)

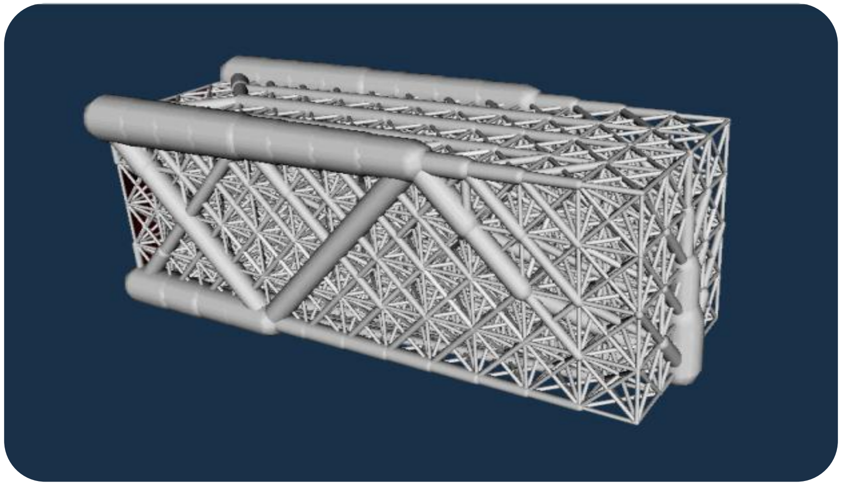
Possibilities: retrofit design

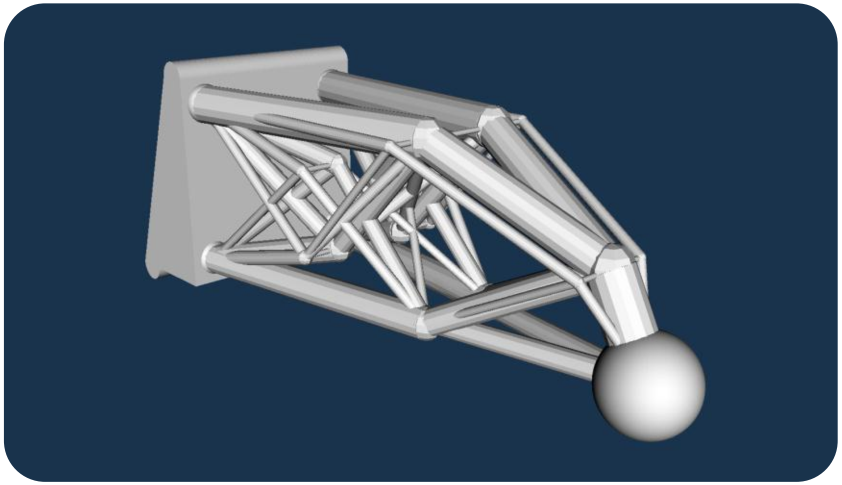
- Example - strengthen arch to carry large load:

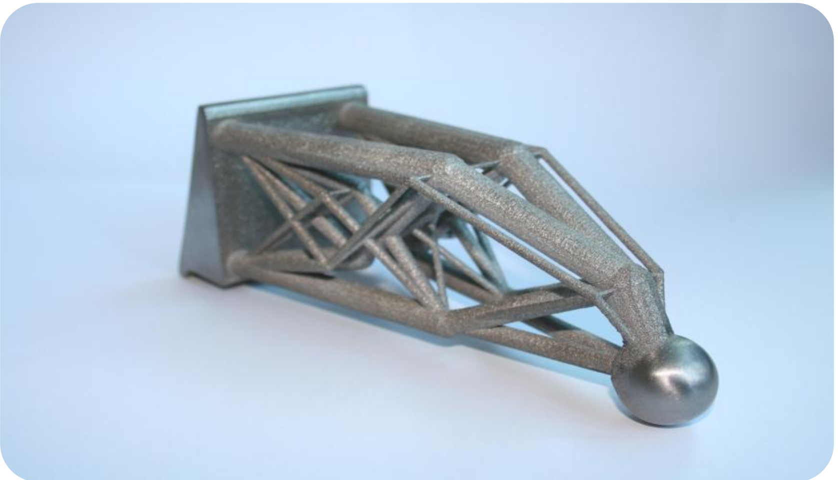


Possibilities: additive manufacture





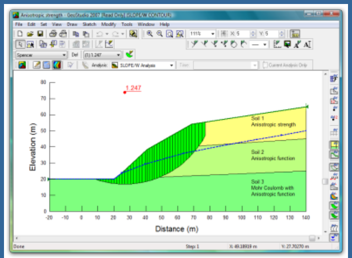




Example II: collapse analysis of continua

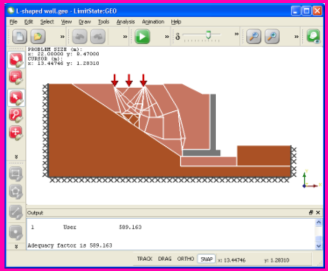
Existing tools for collapse analysis

‘Traditional’: based on hand analysis solutions etc.



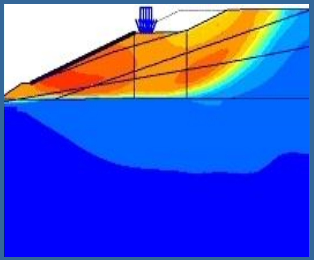
(potentially embedded in simple programs / spreadsheets etc.)

‘Mainstream’: based on computational <u>limit analysis</u>?



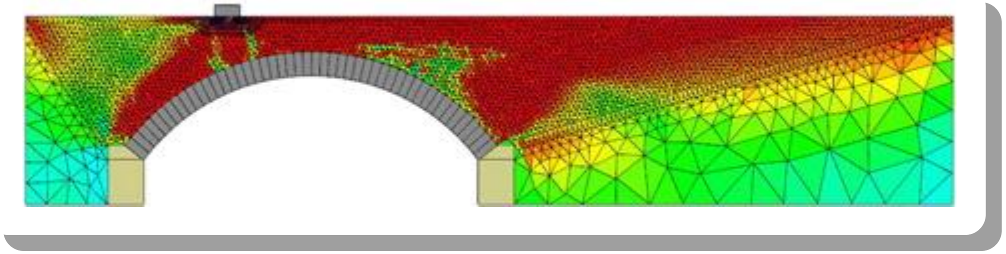
More:
- complex
- time consuming
- input parameters
- expertise required
- accurate [potentially at least!]

‘Advanced’: based on non-linear finite elements etc.



Limit analysis + finite elements

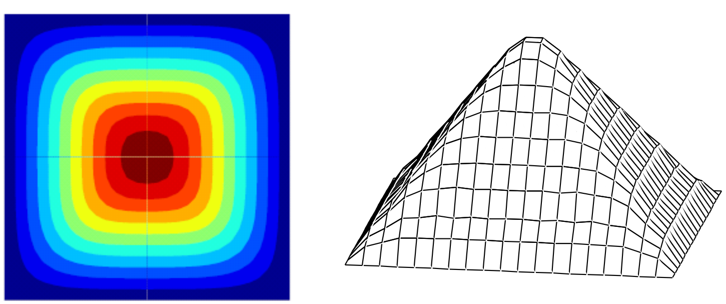
- Involves only strength parameters
- Powerful and flexible, but:
  - needs tailored meshes or high order elements accurate results OR adaptive refinement
  - output arguably lacks clarity of classical ‘hand’ based limit analysis:



  - over 40 years since first paper in this field (Belytschko & Hodge, J. Appl. Mech. ASME, 1967), but still not widely used in industry…

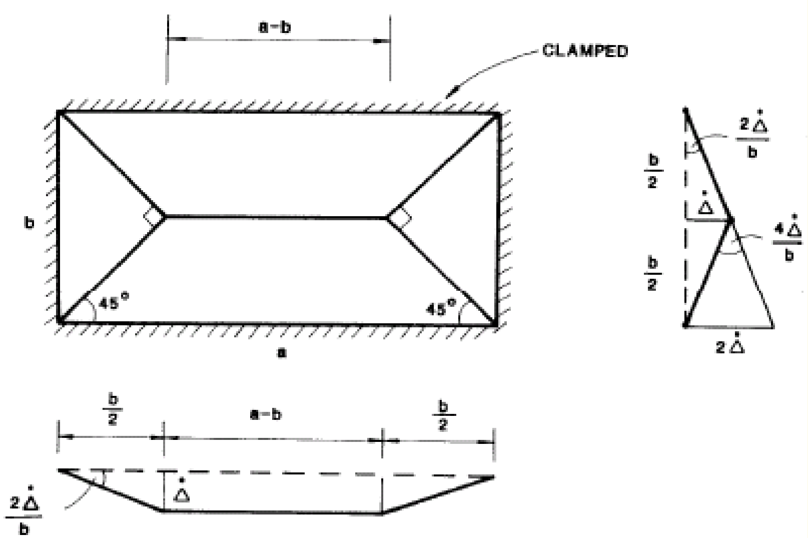
Alternative I: fully continuous

- Meshless methods have recently been explored:
  - Similar to finite element limit analysis but can overcome mesh problems
  - But somewhat complex and strict bounds not available (e.g. EFG plates: Le, Gilbert & Askes. IJNME 2009, 2010)

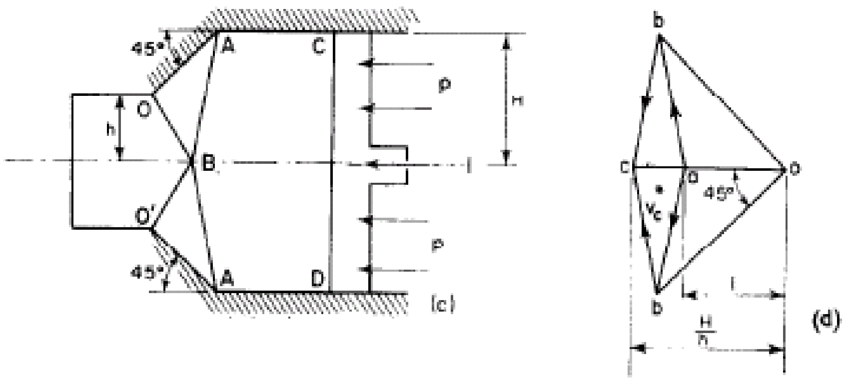


Alternative II: fully discontinuous

- E.g. can we just automate traditional 'hand' type analysis tools for continuum problems?



(Out-of-plane)

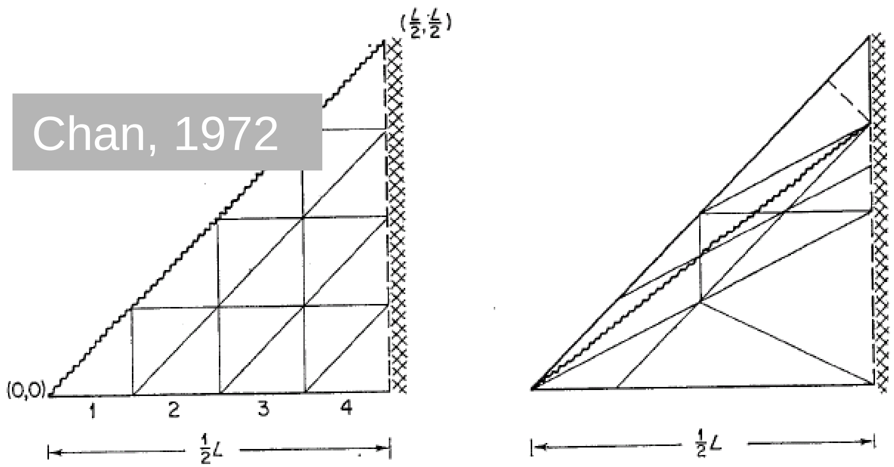


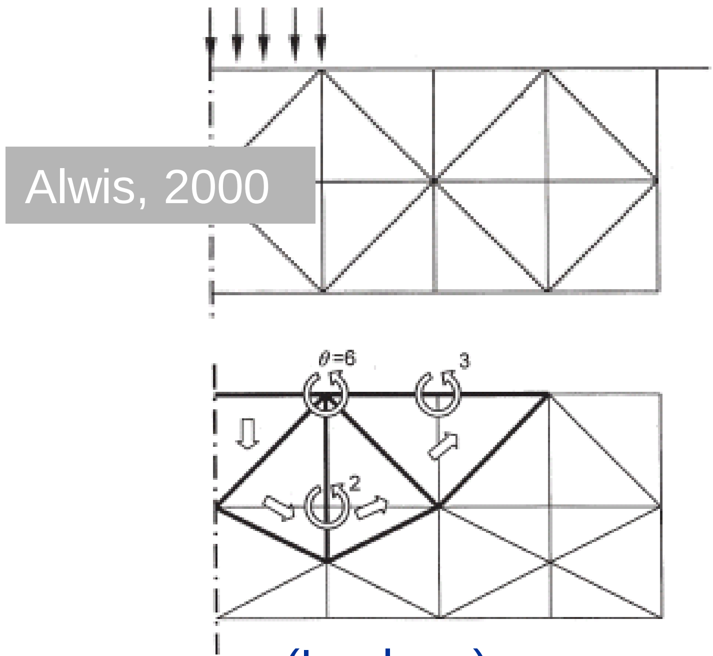
(In-plane)

The University Of Sheffield.

Alternative II: fully discontinuous

- Rigid element based formulations have been tried:



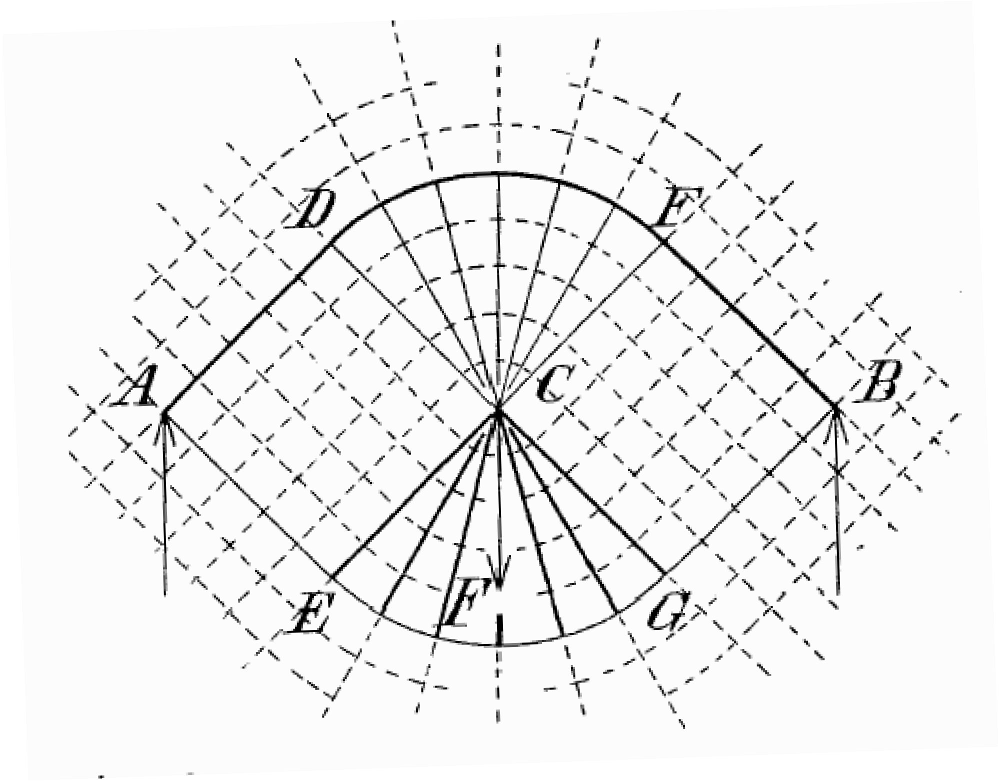


(Out-of-plane)

(In-plane)

- But solutions highly dependent on element topology...

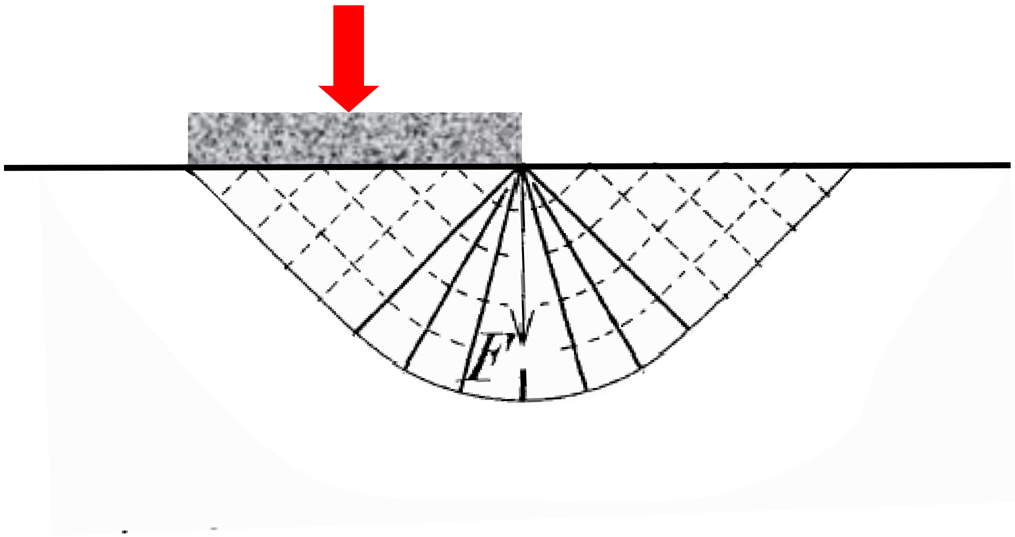
But lets rewind to Michell's seminal 1904 work…



- Slip lines in plane Tresca bodies & optimal 'Michell' trusses both comprise 'Hencky-Prandtl' nets
  - Orthogonal curvilinear co-ordinate systems
  - Analogy discovered by Hemp, Prager in 1950s (& me in 2000s!)

⇒ Should be possible to use layout optimisation for analysis problems…

But lets rewind to Michell's seminal 1904 work…



- Slip lines in plane Tresca bodies & optimal 'Michell' trusses both comprise 'Hencky-Prandtl' nets
  - Orthogonal curvilinear co-ordinate systems
  - Analogy discovered by Hemp, Prager in 1950s (& me in 2000s!)

⇒ Should be possible to use layout optimisation for analysis problems…

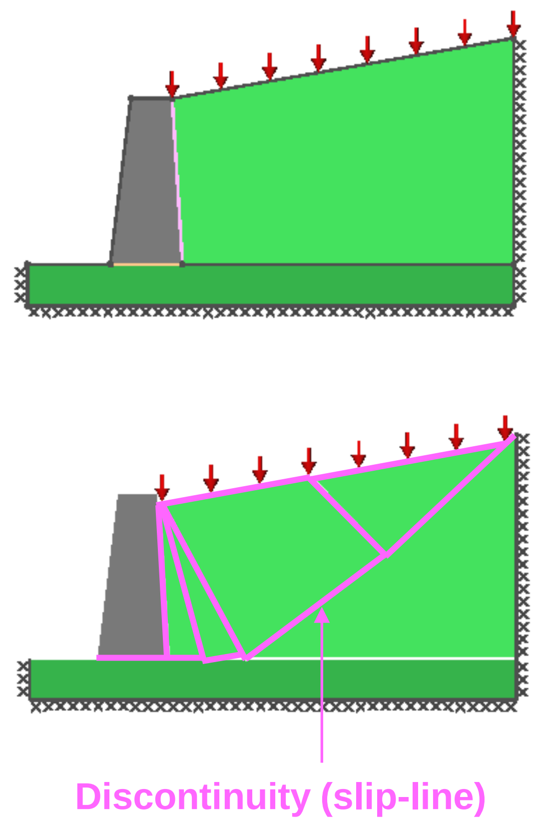
The analogy:

	Truss problem	**Slip-line discontinuity problem**
Problem variables	Internal bar forces	Slip displacements
Governing coefficient matrix	Equilibrium	Compatibility
Applied loads / displacements	External loads	Nodal displacements
Objective function	Minimise volume	Minimise work
Graphical analysis method	Maxwell force diagram	Hodograph (velocity diagram)

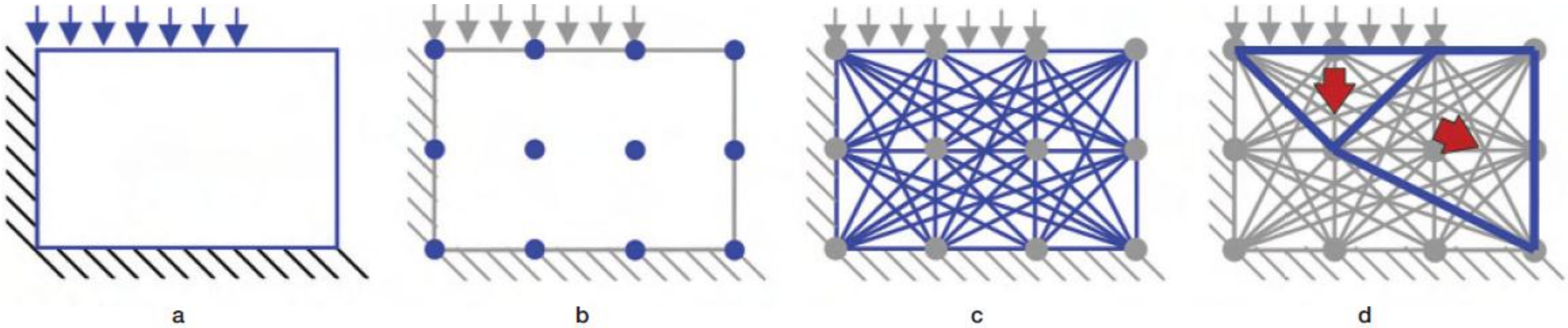
Discontinuity Layout Optimization (DLO)

- Developed fairly recently (Smith & Gilbert, Proc. R. Soc. A. 2007)
- Key = problem formulated in terms of lines of **discontinuity** (rather than rigid elements)
- The critical **layout** of these lines can be determined using **optimization** techniques

⇒ **'discontinuity layout optimization'**



How does DLO work: conceptual



How does DLO work: mathematics

Load factor — internal dissipation

$$\min \lambda \mathbf{f}_{\mathrm{L}}^{T}\mathbf{d} = -\mathbf{f}_{\mathrm{D}}^{T}\mathbf{d} + \mathbf{g}^{T}\mathbf{p}$$

Subject to:

dead load work

$$\mathbf{B}\mathbf{d} = \mathbf{0}$$ ← nodal compatibility

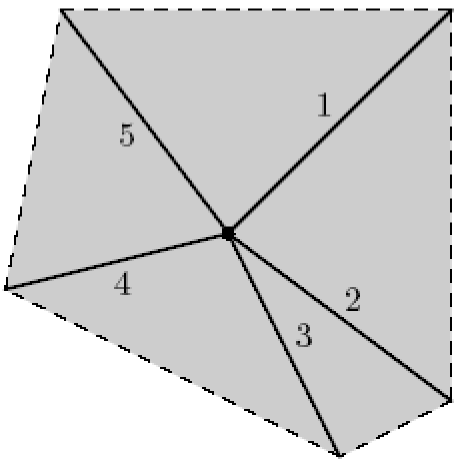
$$\mathbf{N}\mathbf{p} - \mathbf{d} = \mathbf{0}$$ ← flow = f(material model)

$$\mathbf{f}_{\mathrm{L}}^{T}\mathbf{d} = 1$$ ← unit live load work

$$\mathbf{p} \geq \mathbf{0}$$

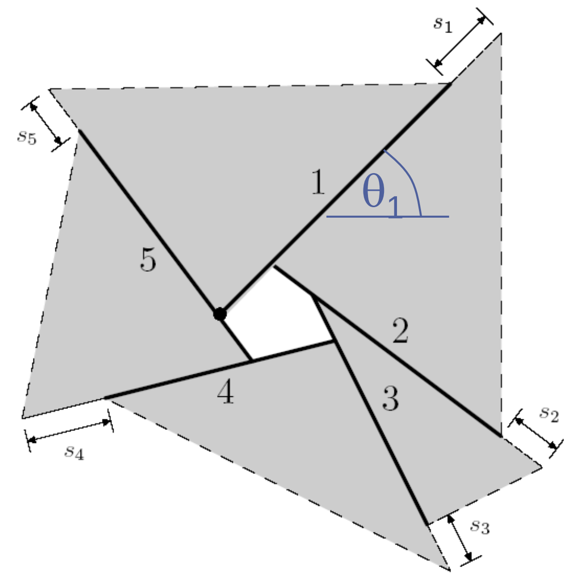
Variables: displacements in **d** and plastic multipliers in **p**

Nodal compatibility



$$\sum_{i=1}^{5} s_i \cos\theta_i = 0$$

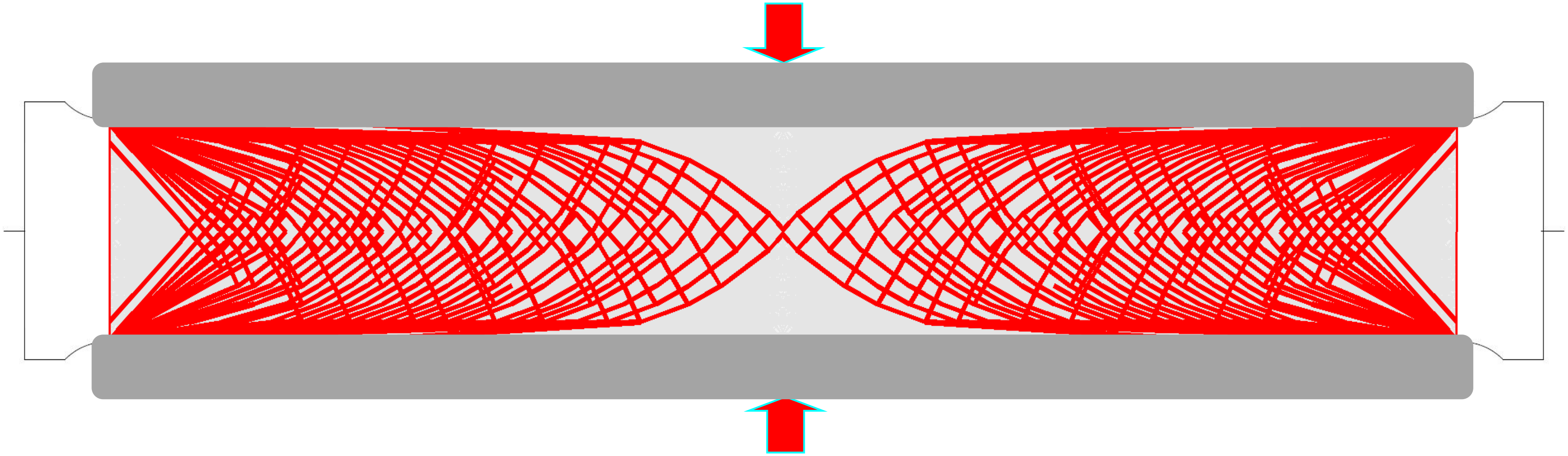
$$\sum_{i=1}^{5} s_i \sin\theta_i = 0$$



The University Of Sheffield.

Implementation: MATLAB

- ≈150 line script for simple plane strain problems at: http://cmd.shef.ac.uk/dlo



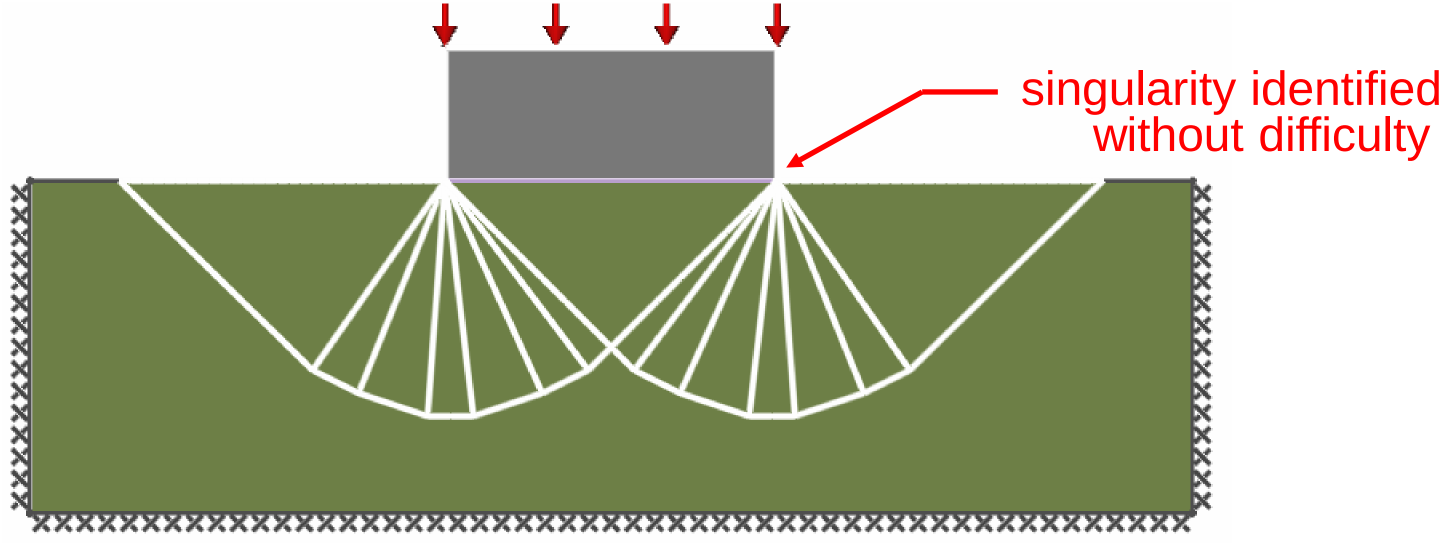
Implementation: industry tool

- Required developments (cf. MATLAB):
  - Multiple domains of general (non-convex) shape
  - Visualization of failure mechanisms
  - Free-body diagrams
  - Water pressures, etc, etc...
- Status:
  - 'LimitState:GEO' launched in 2008 - now widely used across the world
  - Freely available for academic use (see: http://www.limitstate.com/geo)

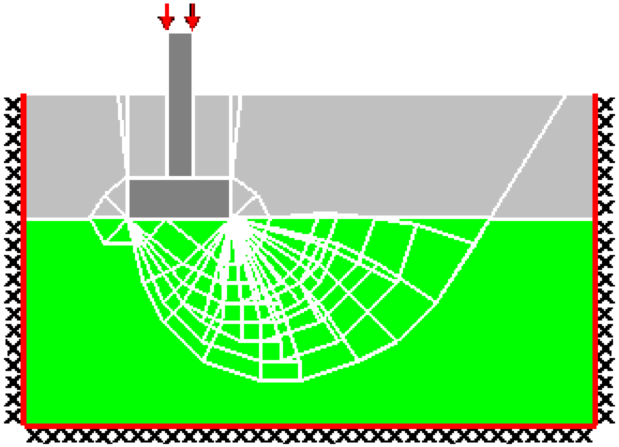
The University Of Sheffield.

Simple DLO example

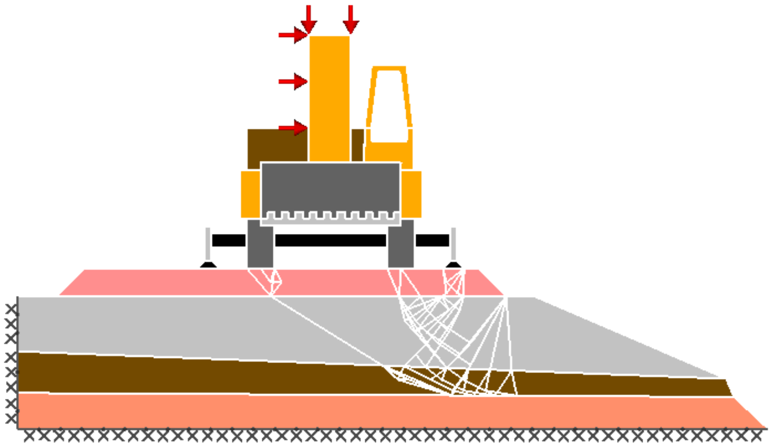
- For ‘Prandtl punch’ problem, solution within 1% of exact solution (2+π) in approx. 1 second:



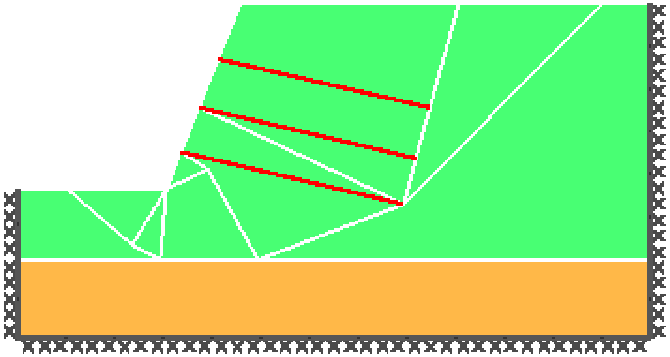
(>100 other benchmarks available at limitstate.com/geo/validation)



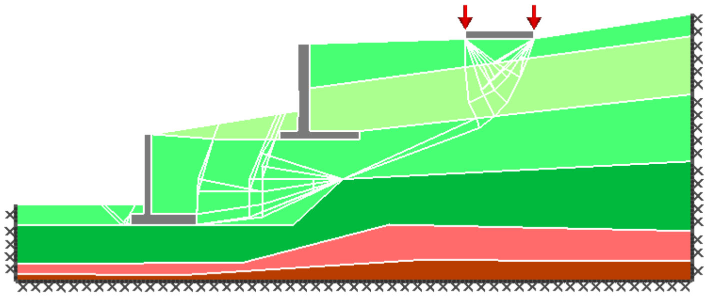
Footings



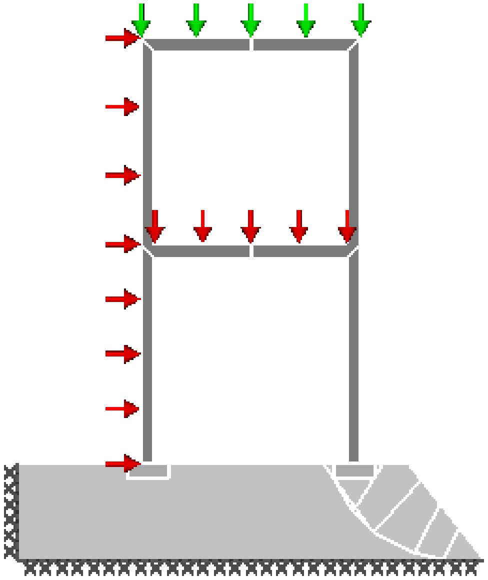
Plant loading



Reinforced slopes



‘Combined’



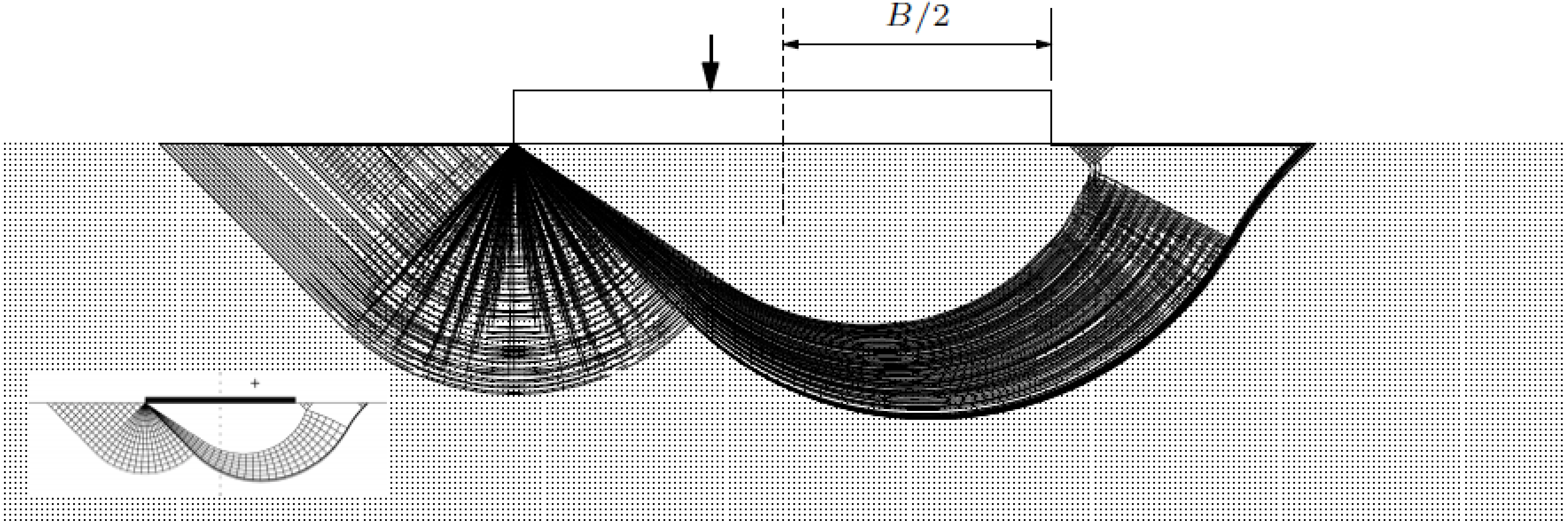
Soil-structure interaction


Civil, Structural and Geote
www.limitstate.com
Salesforce
CMD (internal)
Home - Trac
LS Mail
GMail
CRM
CSE Staff Handbook...
Novel Computation...
Other bookmarks
limitstate
analysis & design software for engineers
Login | Search
Select Language
Home
About
Products
Download
Purchase
Services
Client Area
Geotechnical Analysis
The latest version of LimitState:GEO is available to download now with a wide range of new features and enhancements.
Download now
Find out more
v3.0 Out Now
Analysis & Design Software for Engineers
LimitState specialize in the development of powerful yet easy-to-use software applications which use unique technology to rapidly identify critical collapse mechanisms and associated margins of safety. This allows engineers to move beyond simple 'automated hand calculations' and predefined mechanisms - but without the need to resort to significantly more complex and potentially cumbersome techniques (e.g. non-linear finite element analysis).
Our current software product range includes LimitState:RING, a rapid analysis tool for masonry arch bridges, and LimitState:GEO, a highly innovative geotechnical software product. All software is fully validated and is available for either short or long term use. LimitState also offers customers with full support for its software products, additionally providing informative seminars and bespoke training as required.
Selected Clients:
ARUP
Network Rail
mouchel
AECOM
Interserve Plc
Golder Associates
SNCF
LATEST
NEWS
EVENTS
LimitState software now used by customers in 30 countries
2 Jan 13
LimitState directors to present on Eurocode 7 and limit analysis at IStructE...
3 Jan 13
Merry Christmas & Happy New Year from LimitState
21 Dec 12
Offshore Companies expand usage of LimitState:GEO
7 Dec 12
LimitState:GEO 3.0 Out Now
27 Nov 12
Smoltczyk & Partner GmbH license LimitState:GEO
12 Nov 12
Swanton Consulting license LimitState:RING
18 Oct 12
follow
More
RSS
Legal & Privacy policy | Site map
© LimitState Ltd | Tel: +44 (0)114 224 2240 | E-mail: info@limitstate.com
Hosting by Memset Dedicated Servers

Current work

Current work: DLO + rotations

- Already considered rotations occurring at boundaries (Gilbert et al. Proc. ICE EACM, 2010)
- Now modelling arbitrary rotations with curved slip-lines
- Results look good (Smith & Gilbert, Geotechnique, submitted) :



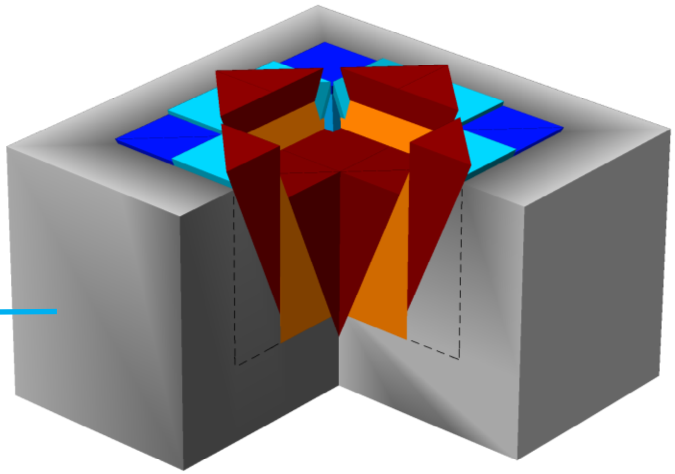
The University Of Sheffield.

Current work: 3D DLO

- Using 'edge based' formulation and <u>triangular slip-surfaces</u>
- Can now obtain solutions using Second Order Cone Programming
- Computationally costly, though reasonable results often obtainable with coarse discretizations, e.g. 3D bearing capacity:

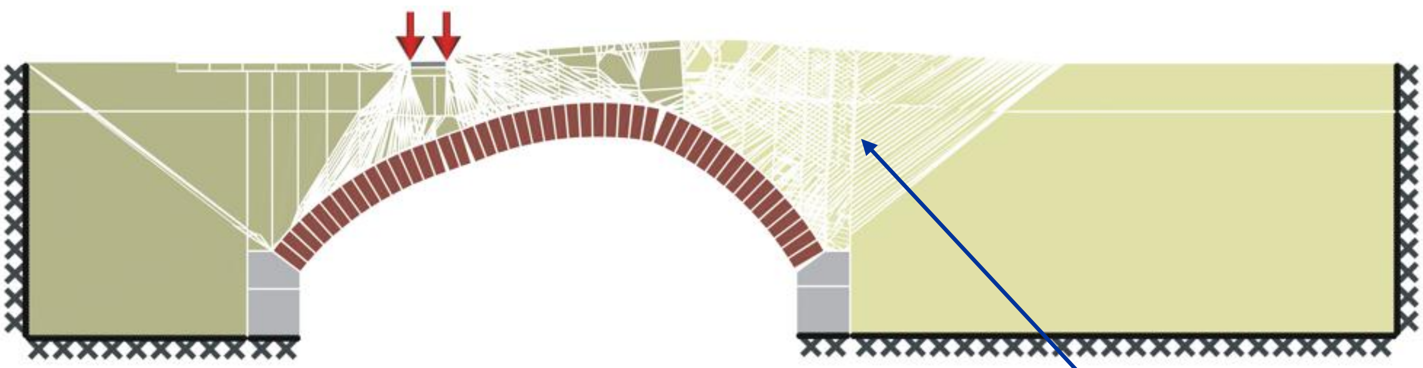
	Bearing Capacity Factor		
Reference	Other	Lower bound	Upper bound
Skempton [2]	6.17[a]		
Gourvenec et al. [3]			6.41
	5.91[b]		
Michalowski [4]			6.56
Salgado et al. [5]		5.52	6.22
Present study			6.42

[a] Empirical
[b] Elasto-plastic finite element



Current work: soil-structure interaction

- Approx. 40% of UK bridge spans are masonry
- Up to 90% of load carrying capacity due to presence of soil backfill
- Limit analysis can be used to model soil and structure, but some difficulties arise:

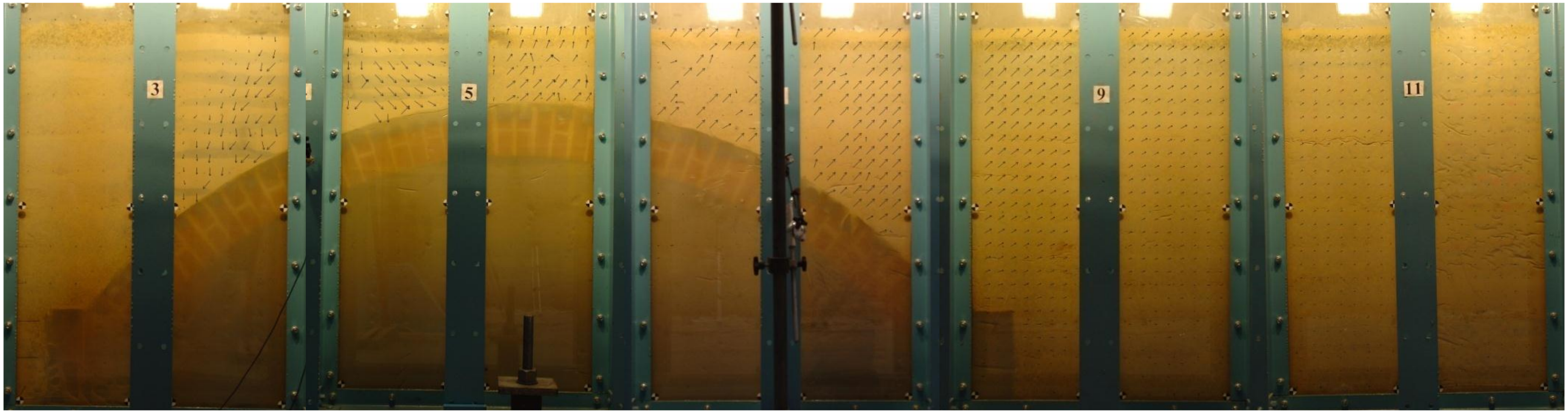


(Gilbert et al. Proc. ICE EACM, 2010)

Peak soil strength **not** mobilised here

Current work: soil-structure interaction

- Digital imaging and Particle Image Velocimetry (PIV) has potential to give a step-change in understanding



- Aiming to use optimization to automatically correlate model and test data

  (large EPSRC/Network Rail project now underway)

Conclusions

The University Of Sheffield.

Conclusions

- Layout optimization is a powerful tool, for use in analysis and design
- Discontinuity layout optimization (DLO) is a powerful new computational limit analysis procedure
  - Typically involves solution of a linear optimization problem, which is easy to solve
  - Singularities are identified automatically
  - Now widely used in industry, and has the potential to form the basis of future mainstream collapse analysis tools
- DLO has been implemented in a short MATLAB script & also in the LimitState:GEO software - both freely available for academic use

The University Of Sheffield.

Acknowledgements

- People:
  - Colin Smith, Andy Tyas, Wael Darwich, Iain Haslam, Tom Pritchard, Sam Hawksbee, Dong Nguyen, Aleksey Pichugin
- Organizations:
  - EPSRC (UK Government Research Council)

Thank you for listening!