Bubble-scale predictions of how foams flow

Simon Cox,
foams@aber.ac.uk

RIVIC
Research Institute of Visual Computing
P&G
EPSRC
The Institute of Non-Newtonian Fluid Mechanics
PRIFYSGOL ABERYSTWYTH UNIVERSITY

Foam structure

Soap films

Plateau borders (PBs)

Vertices/nodes

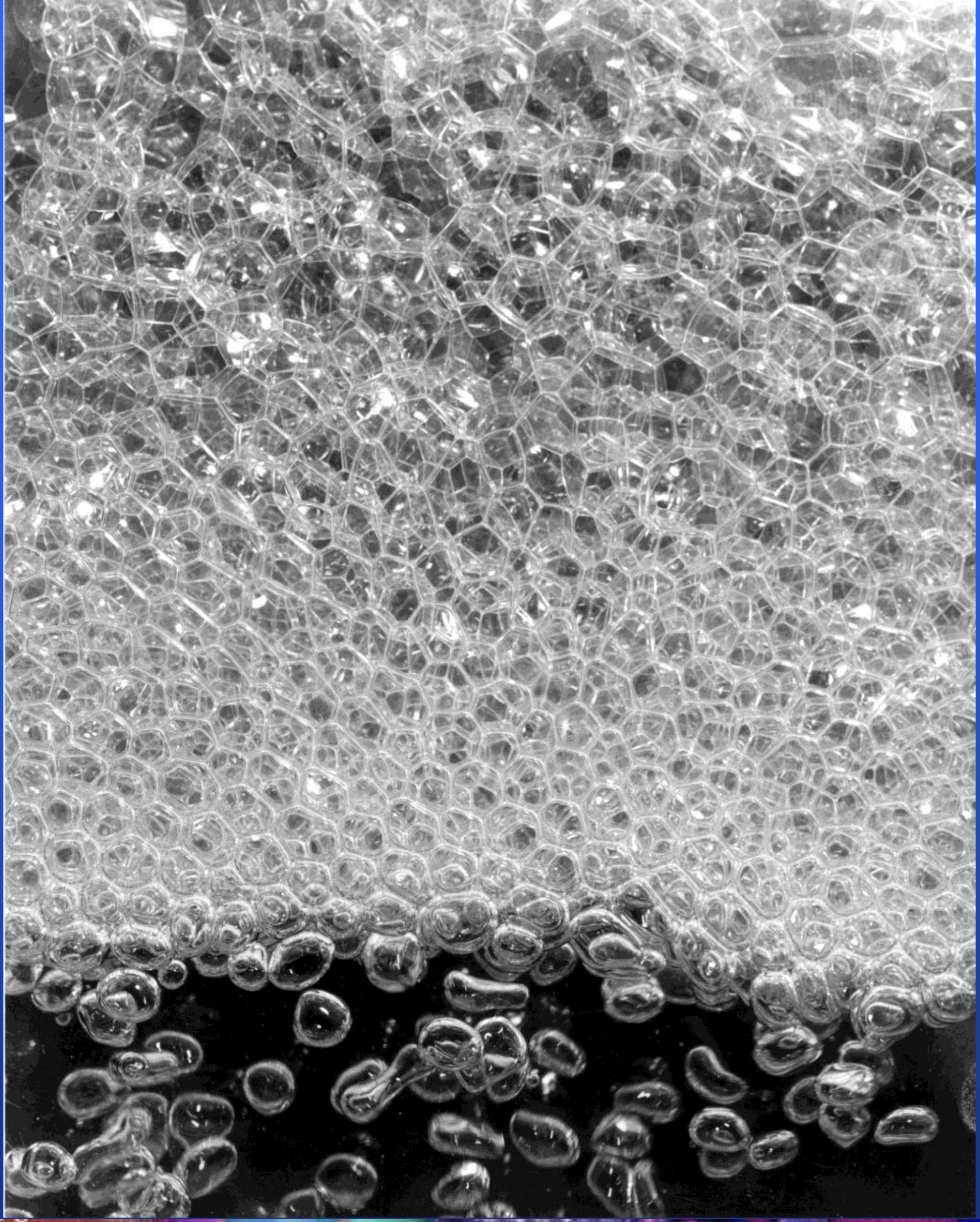
Liquid fraction is

$$\Phi_l = \frac{\text{liquid volume}}{\text{total volume}}$$

Dry foam

Wet foam

Bubbly liquid



Photograph courtesy of J.J. Cilliers (Imperial/Manchester)

Why are foams interesting?

Many applications of industrial and domestic importance:

- Enhanced oil recovery
- Fire-fighting
- Ore separation
- (Industrial) cleaning
- Vehicle manufacture
- Food products

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New Scientist, 25 Feb, 1972

A foam blocks pores in the rock (high yield stress) and the detergent "loosens" the oil (lowers surface tension) to improve recovery efficiency.

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A foam excludes air (oxygen), covering and cooling the (e.g. oil) fire.

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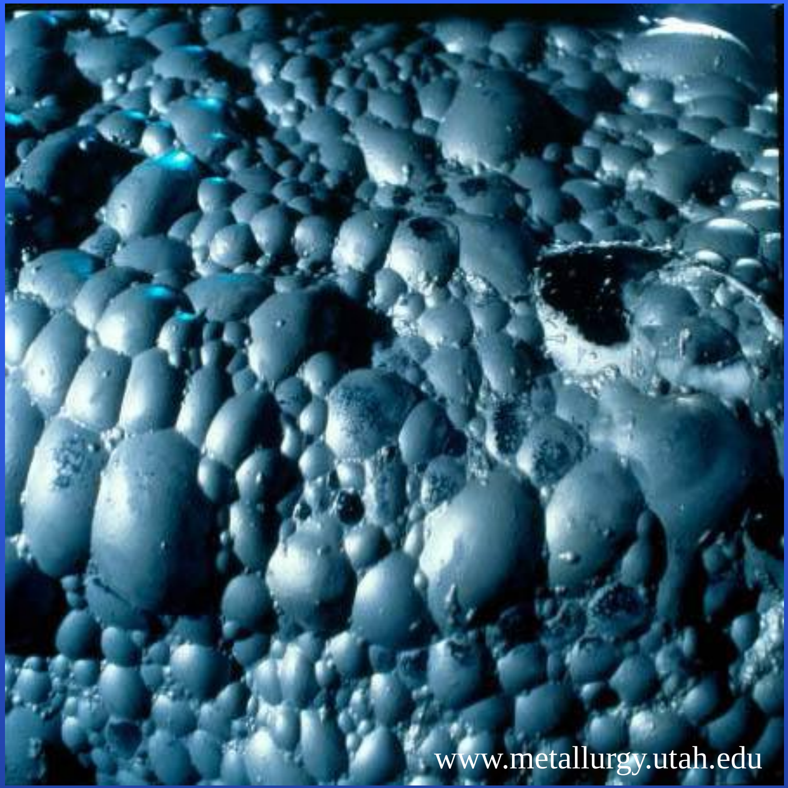

A chemically-tuned foam is used to separate hydrophobic ore (zinc, lead, gold, ...) from hydrophilic rock.

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e.g. decontamination of nuclear fuel vessels, where use of a foam reduces the volume of effluent, and high yield stress dislodges "dirt" more easily.

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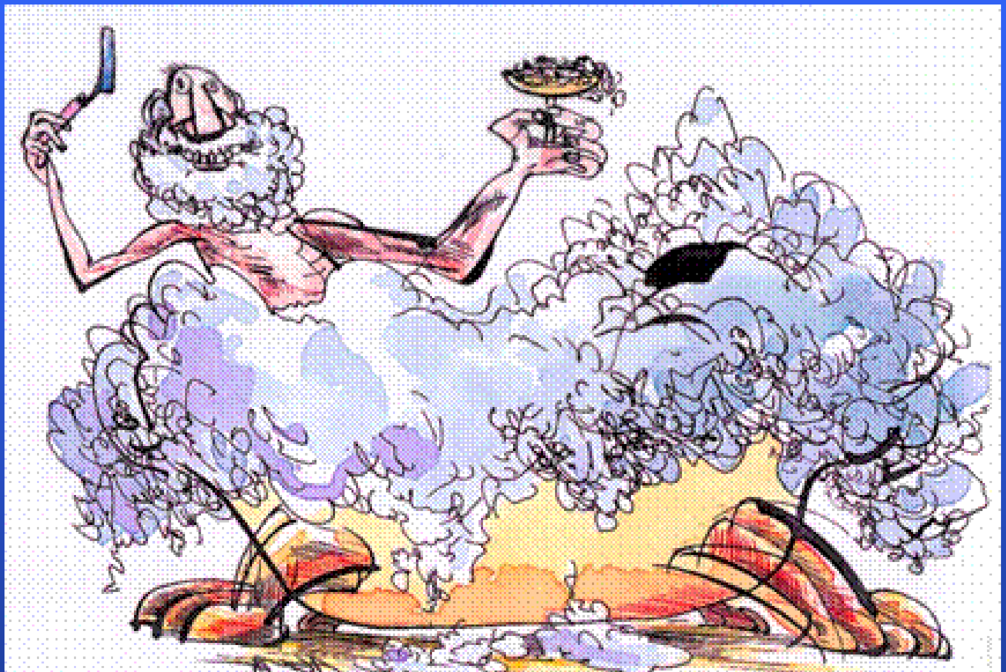

Courtesy of F. Graner

Foam desired by the consumer as a measure of "cleaning power", but often impedes the process

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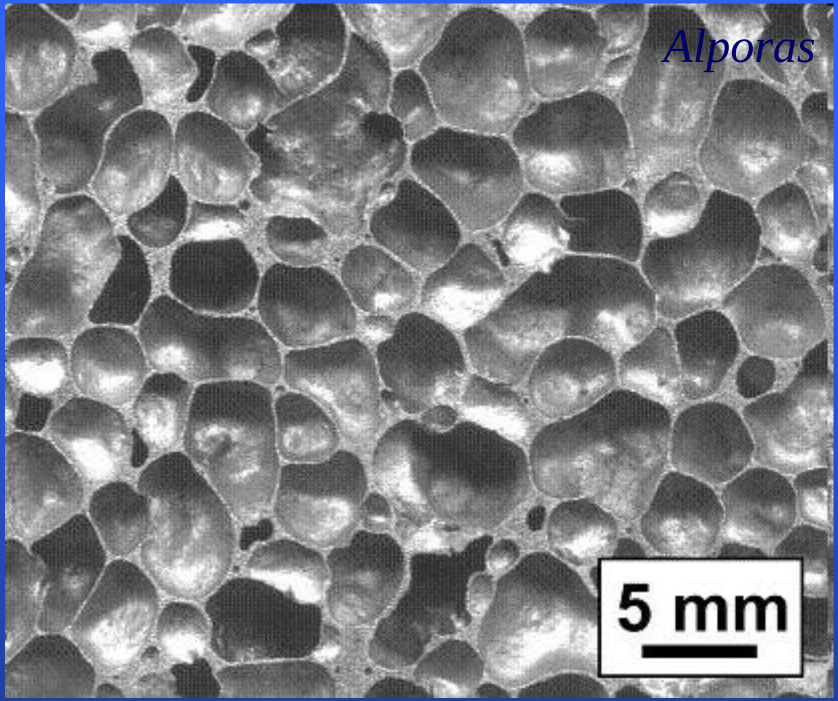

Many solid foams are made from liquid precursors

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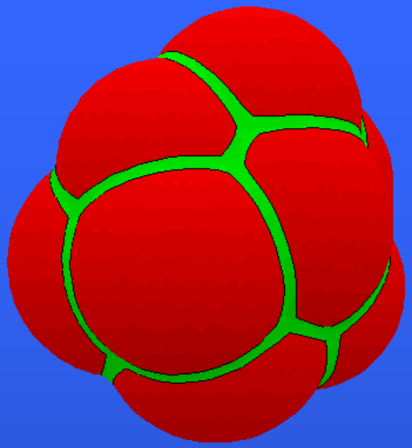
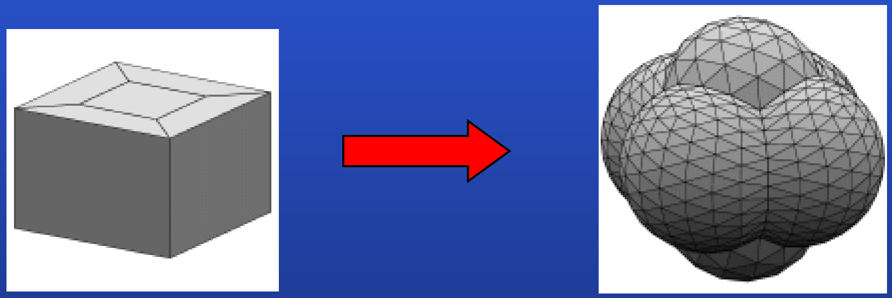
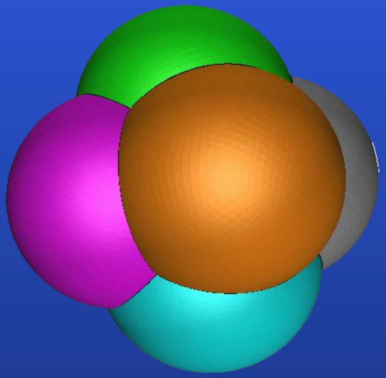
Companies like to sell you air!
Bubbles control texture.
Highly concentrated emulsions are similar to foams.

Simulating foam structure

Ken Brakke's Surface Evolver
for modelling soap bubbles, foams, and other liquid surfaces shaped by minimizing energy subject to various constraints.







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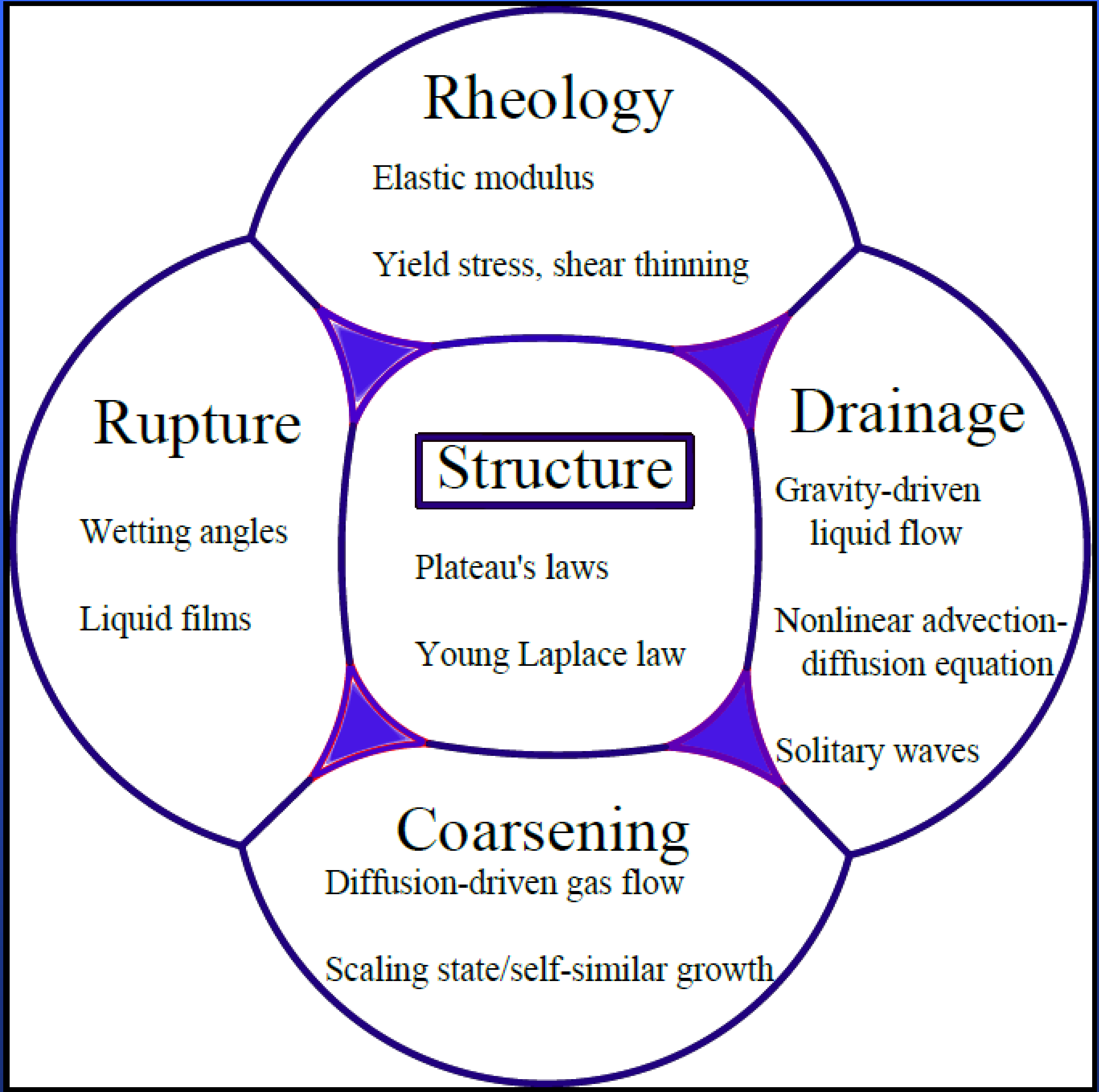

Rheology
Elastic modulus
Yield stress, shear thinning
Rupture
Wetting angles
Liquid films
Structure
Plateau's laws
Young Laplace law
Drainage
Gravity-driven liquid flow
Nonlinear advection-diffusion equation
Solitary waves
Coarsening
Diffusion-driven gas flow
Scaling state/self-similar growth

- Structure: Each soap film is a minimal surface; provide solutions of isoperimetric problems.

- Lifetime: largely dictated by the local static structure
  (e.g. stability, foamability, drainage)

- Flow: How does structure influence rheological response?

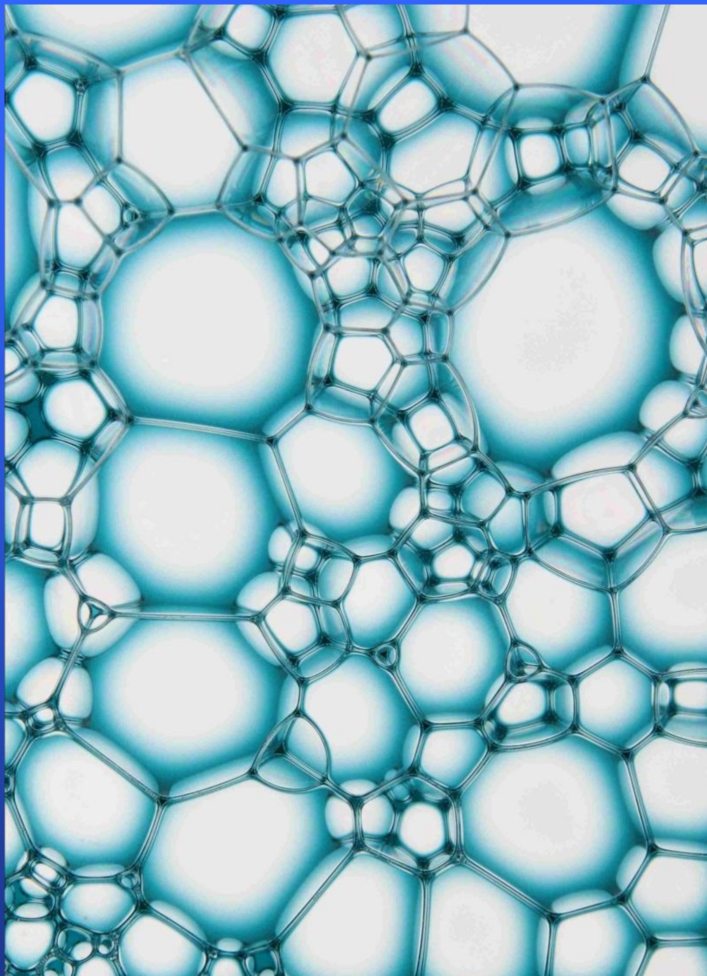
Why is a soap bubble spherical?

(to a good approximation)

A soap film minimizes its surface energy ≡ surface area.

(Isoperimetric problem)

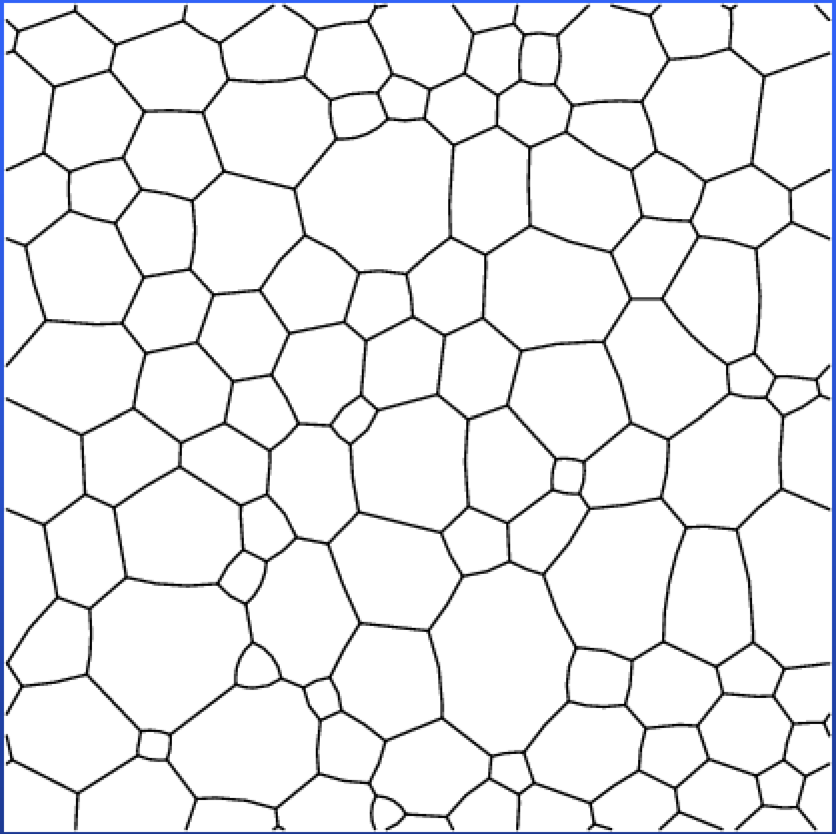
Foam Structure



- An equilibrium dry foam minimizes its surface area at constant volume.
- As a consequence (Plateau, Taylor) it is a complex fluid with special local geometry…
- Films meet **three-fold** at 120º angles in lines (Plateau borders), and the lines meet **tetrahedrally**.
- Laplace Law: film curvatures balanced by pressure differences, so each film has constant mean curvature.

The ideal two-dimensional soap froth

Laplace-Young & Plateau laws:
in a dry foam at equilibrium, each film is a circular arc and they meet three-fold at 120°.



Experimental realization possible, e.g. monolayer of bubbles between parallel glass plates.

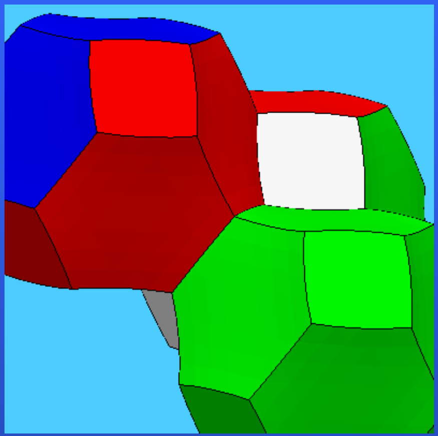
Gives insight into 3D case: allows bubble position and deformation to be seen.

Interesting system in its own right (e.g. microfluidics).

Minimal surface problems

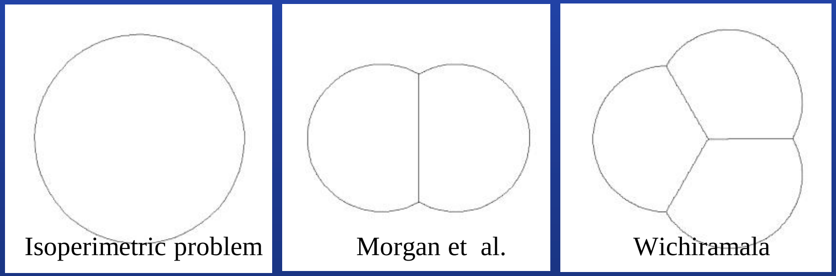
- What is the least perimeter division of the plane into equal area cells? Hexagonal honeycomb (Hales).

- What is the least surface area partition of space into equal-volume cells? Kelvin problem; solution unknown.



- What is the arrangement of N cells of equal area that minimizes the total perimeter?

  Proofs only for $N \leq 3$.



Foams

Pure mathematicians refuse to assume that in the optimal arrangements each bubble consists of one component (connected). If we assume connectedness, could we:

+ enumerate all possible topologies? How?

+ develop numerical search algorithms?

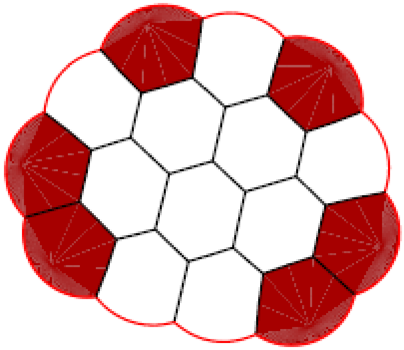
+ just test likely structures?

Finite clusters with free boundary

- What is the arrangement of N bubbles of area $A = \frac{3\sqrt{3}}{2}$ that has least perimeter?
- Can we expect the periphery to have any particular shape? Should it be as circular as possible?
- Or does the arrangement maximise the number of hexagons?
- Should any non-hexagonal "defects" be well-separated?

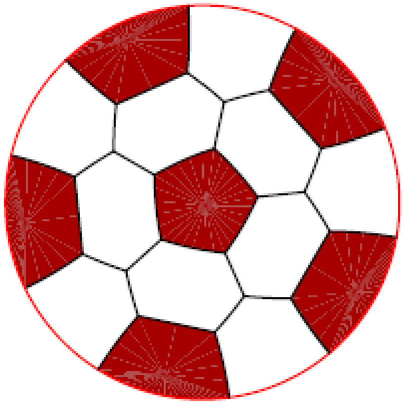
$N = 16$

60.235

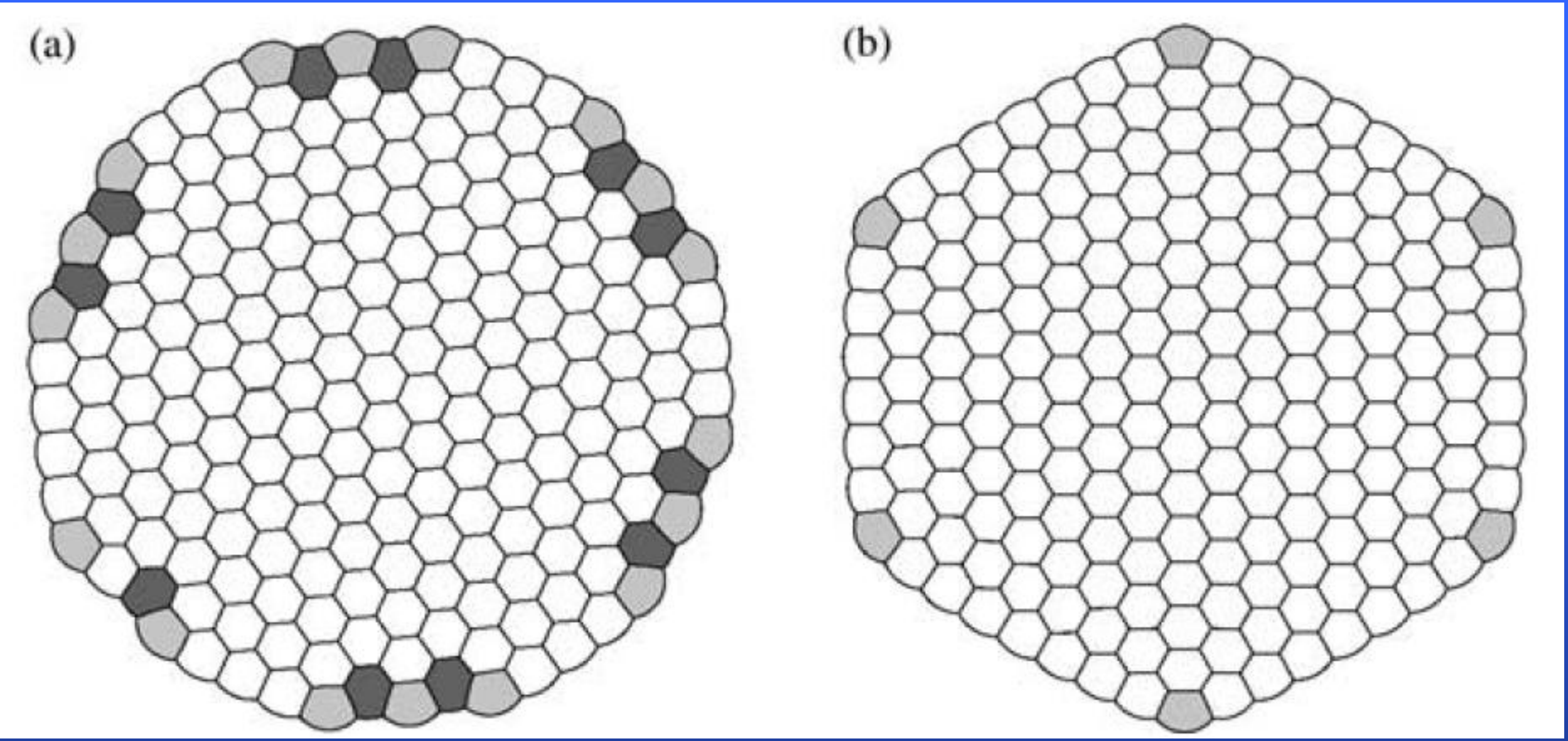


$N = 16$

61.415



Finite clusters with free boundary: N = 200

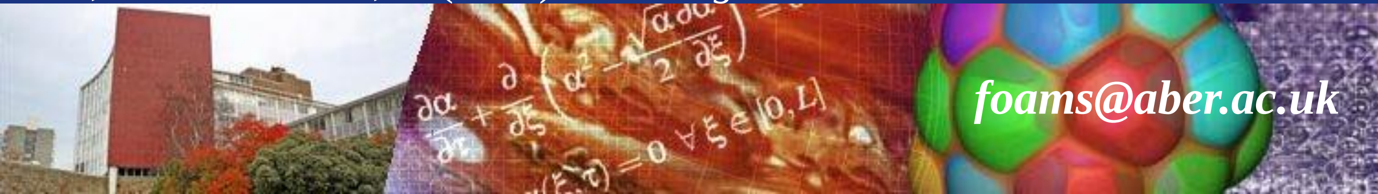


Circular region cut from hexagonal lattice

Honeycomb-like structure pinned to hexagonal lattice ...

with lower perimeter

Cox, S.J. and Graner, F. (2003) *Phil. Mag.* **83**:2573.



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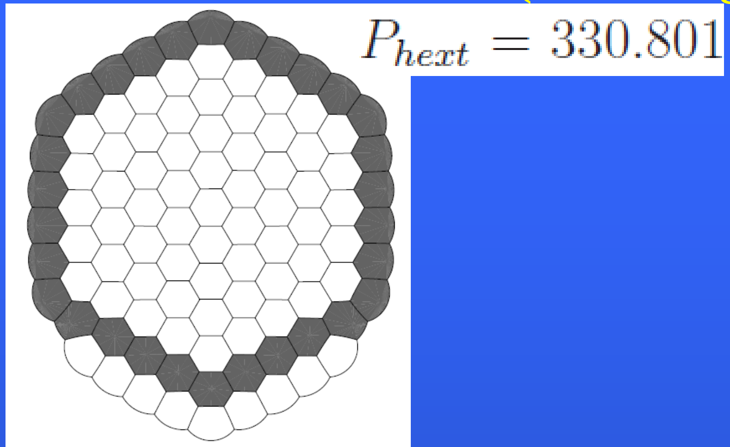
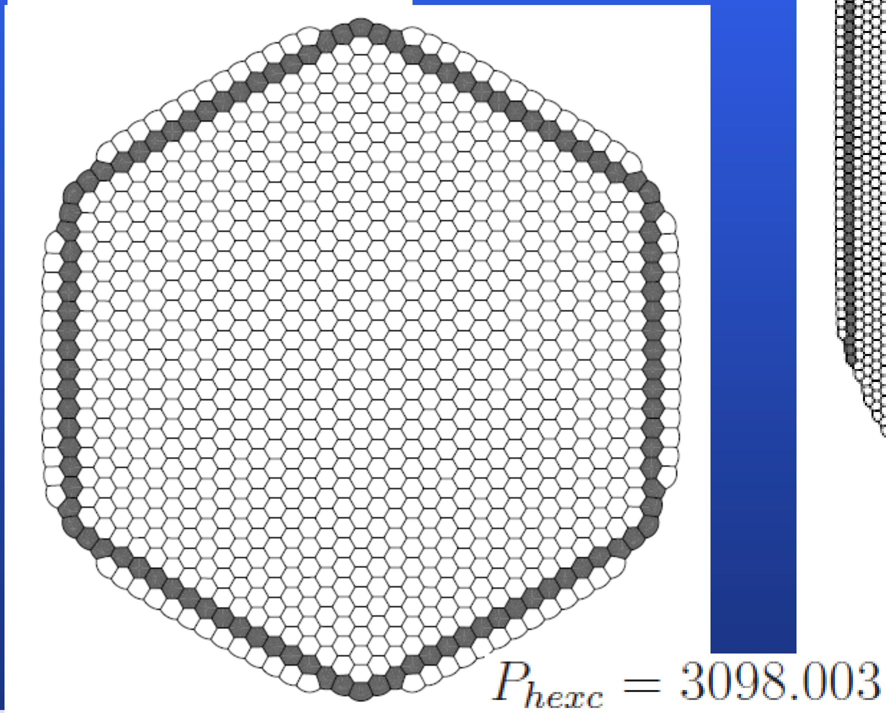
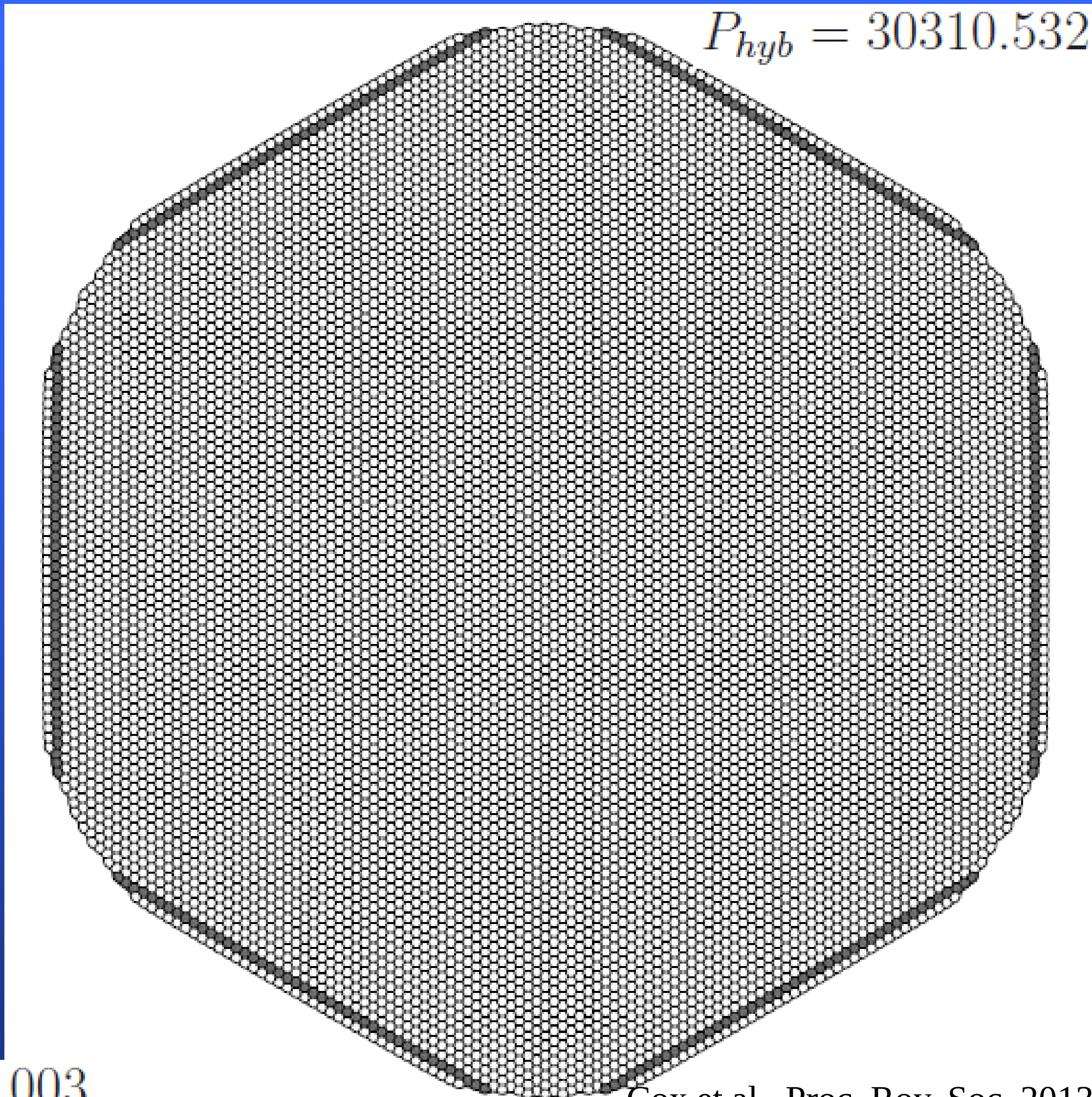
Candidate minima (challenge problems?): N=100, 1000, 10000



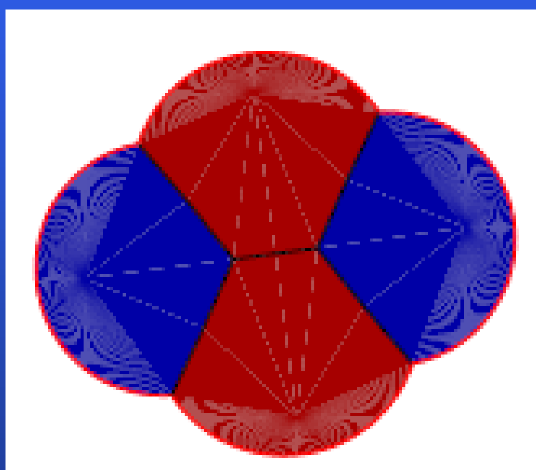


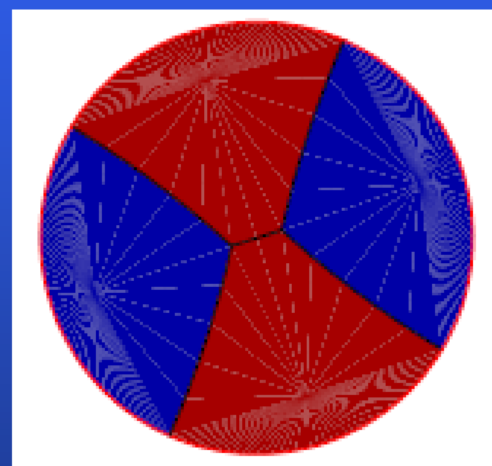


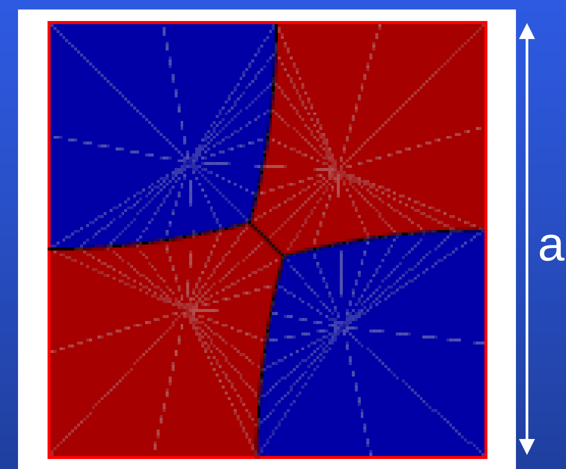
Cox et al., Proc. Roy. Soc. 2013

Isoperimetric problems in foams

Having found an optimal candidate for the free case, for which fixed **boundary shapes** does it remain optimal?







internal perimeter = 1.976a

films meet walls at 90° instead of 120°

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- Structure: Each soap film is a minimal surface; provide solutions of isoperimetric problems.

- Lifetime: largely dictated by the local static structure
  (e.g. stability, foamability, drainage)

- Flow: How does structure influence rheological response?

Stability: Coarsening / Ostwald ripening

Gas diffuses across soap films due to pressure differences between bubbles.

In a dry 2D foam, rate of change of area depends upon number of sides *n*:

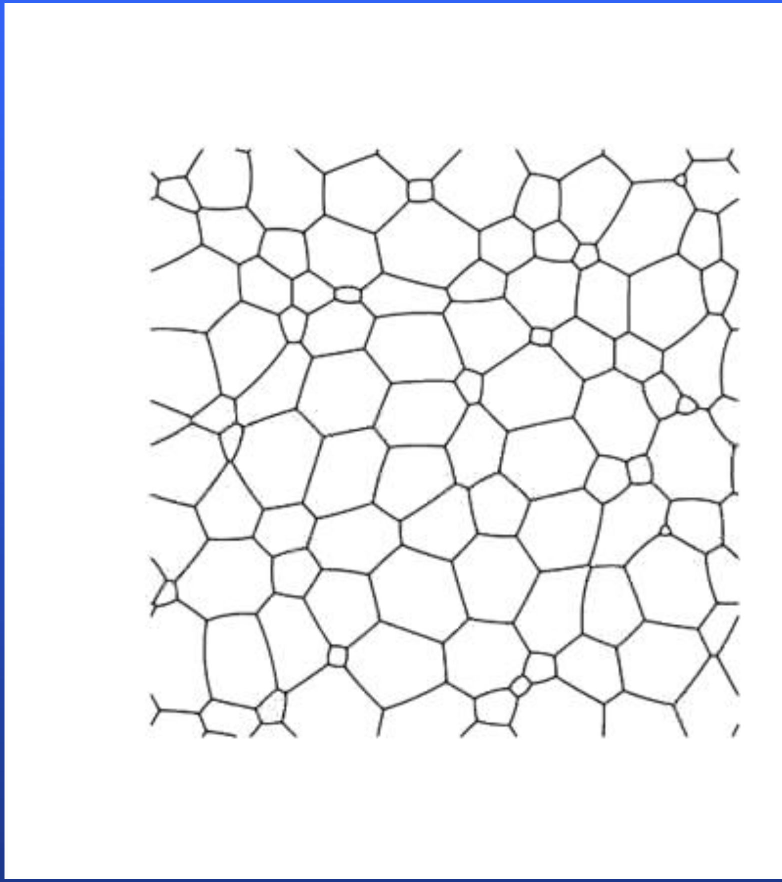
$$\frac{\mathrm{d}A}{\mathrm{d}t} = -2\pi M\gamma\left(1 - \frac{1}{6}n\right)$$

No effect on six-sided bubbles.
Scaling behaviour at long times.
Average bubble area increases.

(Von Neumann, 1952)

Permeability *M* allows rate to be tuned.

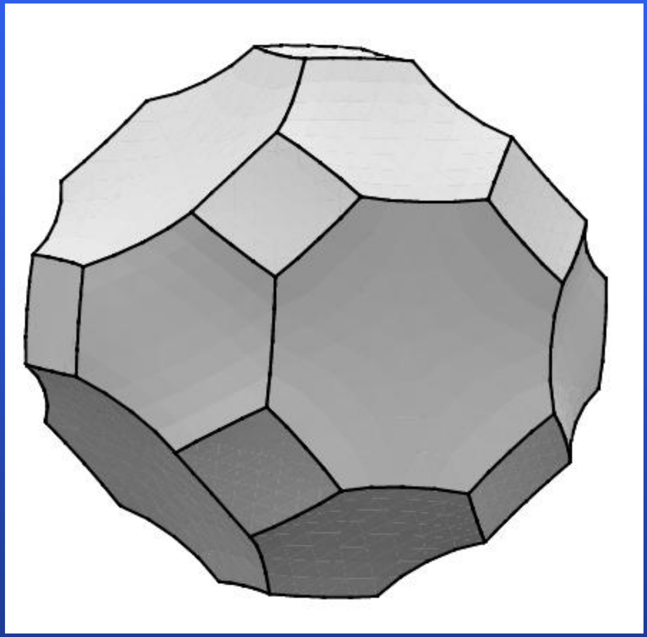
Note curvature of edges induced by angles at vertices:



Stability: Coarsening / Ostwald ripening

In 3D, growth rate does NOT depend only on number of faces ...

… the integral of pressure difference (curvature) over the faces has many contributions.



Probably no invariant bubble.

Probably scaling invariance at long times.

Stability: Collapse / coalesence / rupture

Caused by:
- soil
- impurities
- vibration
- film thinning
- …

Average bubble size increases.

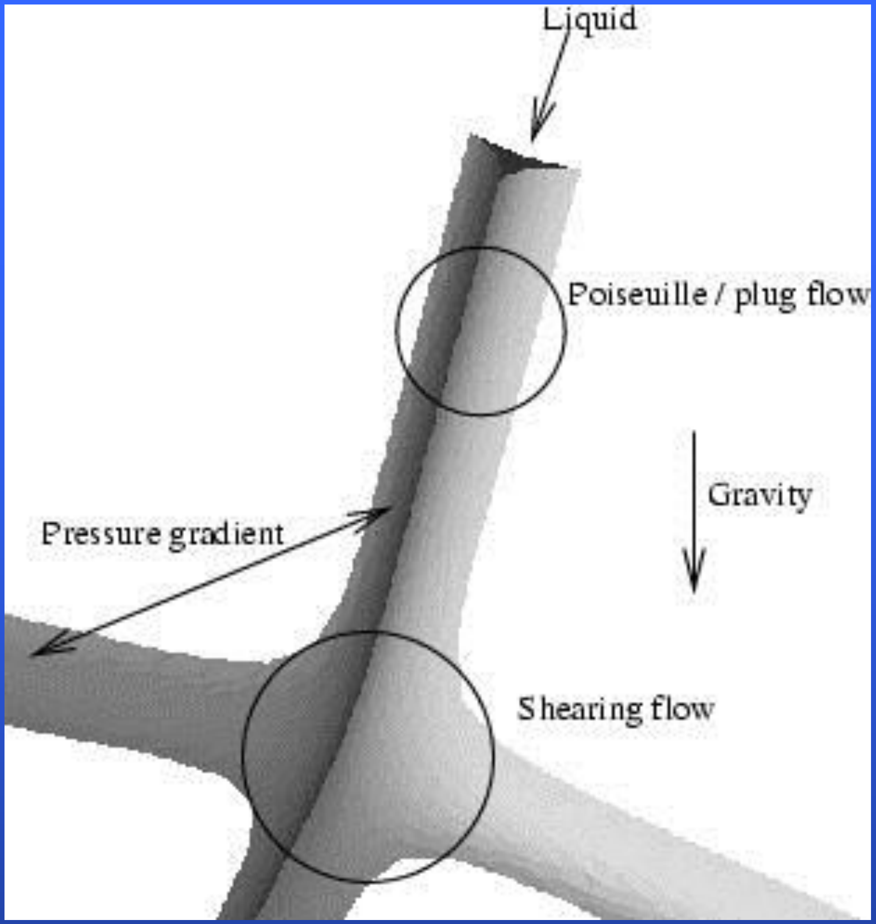
Very hard to predict accurately

Foamability: Liquid drainage

- Plateau border network swells when liquid added (cf porous media).

- Gravity draws liquid downwards, resisted by viscous dissipation and pressure (capillary) forces

- Foam Drainage equations for local liquid fraction based upon cross-sectional area *A* of PBs:



$$\frac{\partial A}{\partial t} + \nabla . \underline{Q} = 0, \qquad e.g.\ \underline{Q} = \frac{1}{\eta^*}\left(\rho g A^2 \hat{z} - C\gamma \nabla^2 A^{3/2}\right)$$

η^* is the effective viscosity, C is a geometric constant, ρ is density and g is gravity

Foamability: Liquid drainage

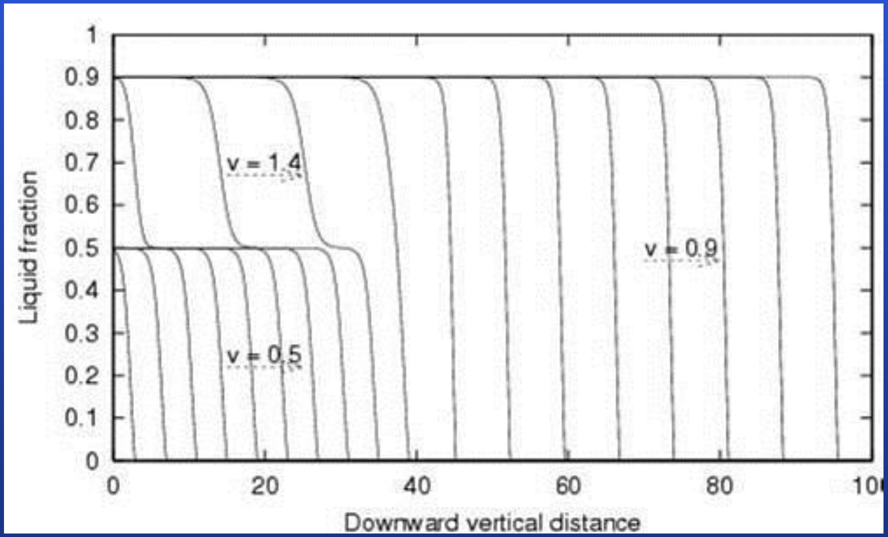
e.g. 1D FDE in non-dimensional form:

$$\frac{\partial \alpha}{\partial t} + \frac{\partial Q}{\partial z} = 0, \quad Q = \left(\alpha^2 - \sqrt{\alpha} \frac{\partial \alpha}{\partial z} \right)$$

- Scaling solutions in some limits (low gravity, low surface tension)

- Travelling wave solution for constant input of liquid ("forced drainage"):

$$\alpha \sim \sqrt{Q_0} \tanh^2 (z - vt)$$



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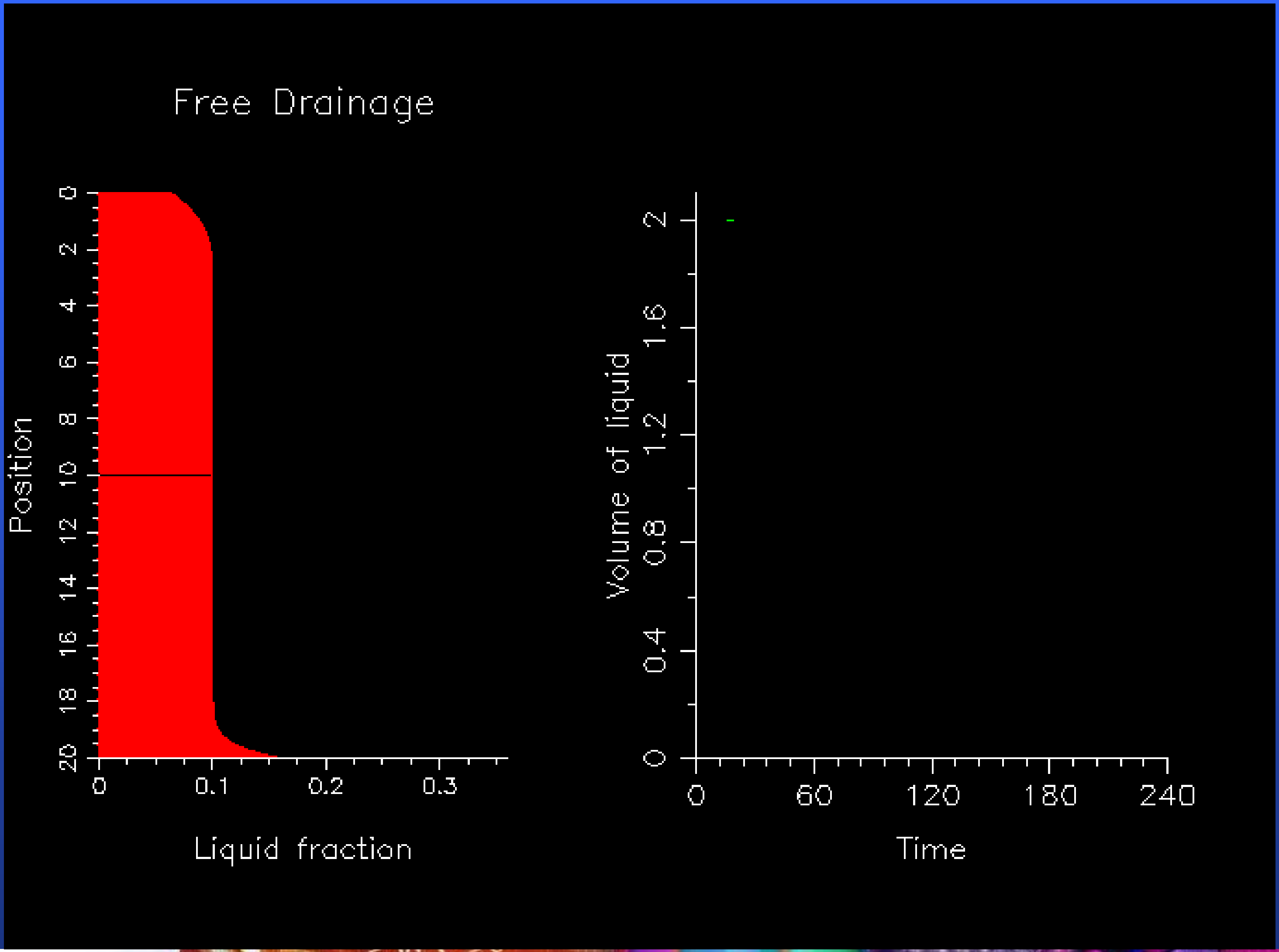
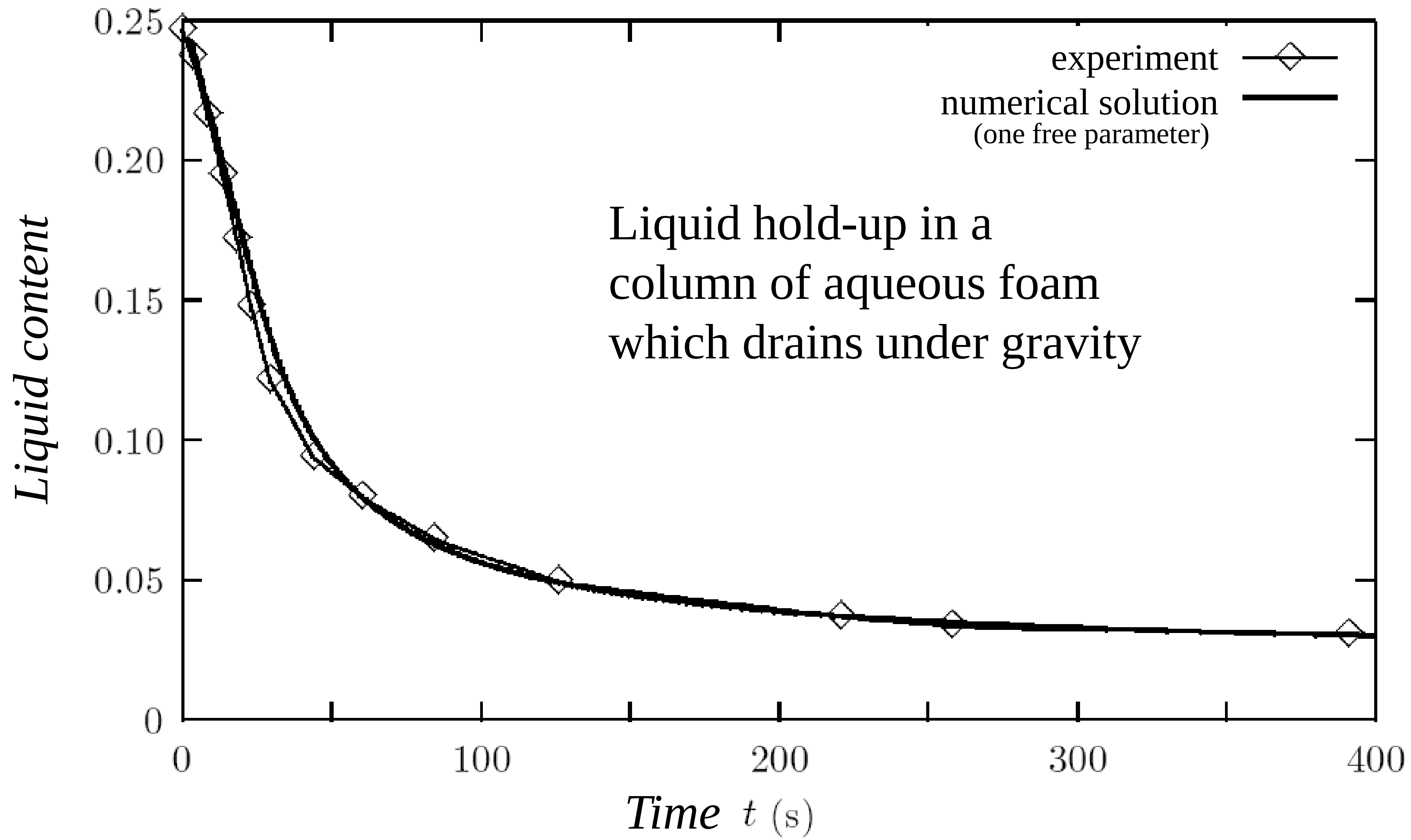
Foamability: Liquid drainage



Foamability: Liquid drainage



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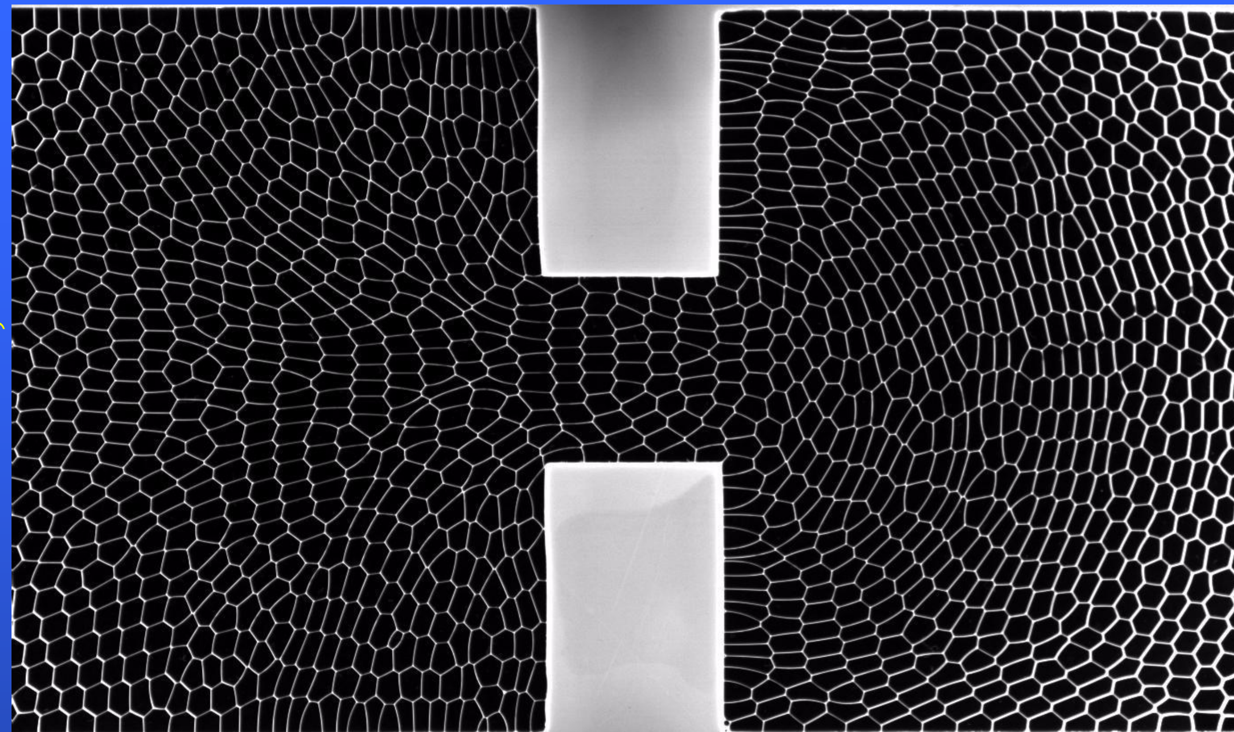
Foam Rheology

Elasticity:
bubbles/films stretch;

Plasticity:
T_1 topological changes: reduce total perimeter and stress; occur even in the zero shear-rate limit;

Viscosity:
e.g. when films move and when new films are formed after T_1s

Sian Jones, J. Rheol. 2012



Average bubble area ~ 0.1 cm^2 (s.d.= 0.03cm^2); commercial detergent; $\Phi_l = 0.008$; Volumetric flow rate ~ 0.03 l/min

- Elastic solid at low strain: $G \sim \gamma / R \, f_1(\Phi_l)$
- Plastic solid as strain (or strain-rate) increases: $\tau_0 \sim \gamma / R \, f_2(\Phi_l)$

Foam as a complex fluid

Shear thinning fluid with a yield stress suggests a Herschel Bulkley model:

$$\tau = \tau_0 + K\dot{\gamma}^n$$

… but many different suggestions for exponent *n*, and no clear relationship between *n* and polydispersity, liquid fraction, …

Instead, predict deformation, flow and effects of confinement based on local bubble structure.

For the ordered (hexagonal) case, see work by Princen.

Now need numerics to explore disordered structure.

Foam rheology

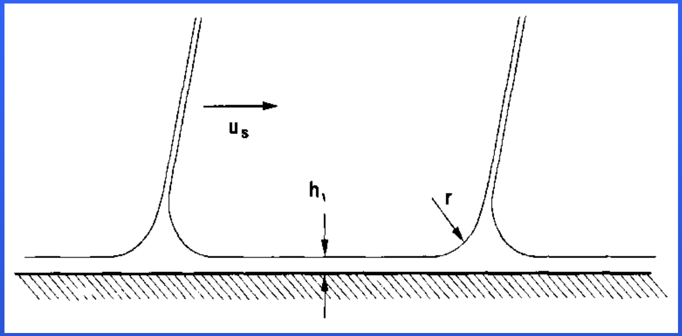
Important time-scales are set by:

- Structural (mechanical) relaxation

- Friction at boundaries

- Bulk and surface viscosities

- Equilibration of surfactant concentration
  (within surface and with bulk)



in addition to coarsening (bubble size) and drainage (liquid fraction)

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Quasi-static simulations of a dry foam

Assume that time-scale of equilibration is faster than shear-rate and gas diffusion.

Foam passes through a sequence of equilibrium configurations.

Idea is to satisfy Laplace-Young Law for each film at each step:

$$\gamma C = \Delta p$$

In practice, minimize total surface area subject to volume constraints:

$$E = \sum_{edges\, i} \gamma L_i + \sum_{bubbles\, j} p_j (A_j - A_{j0})$$

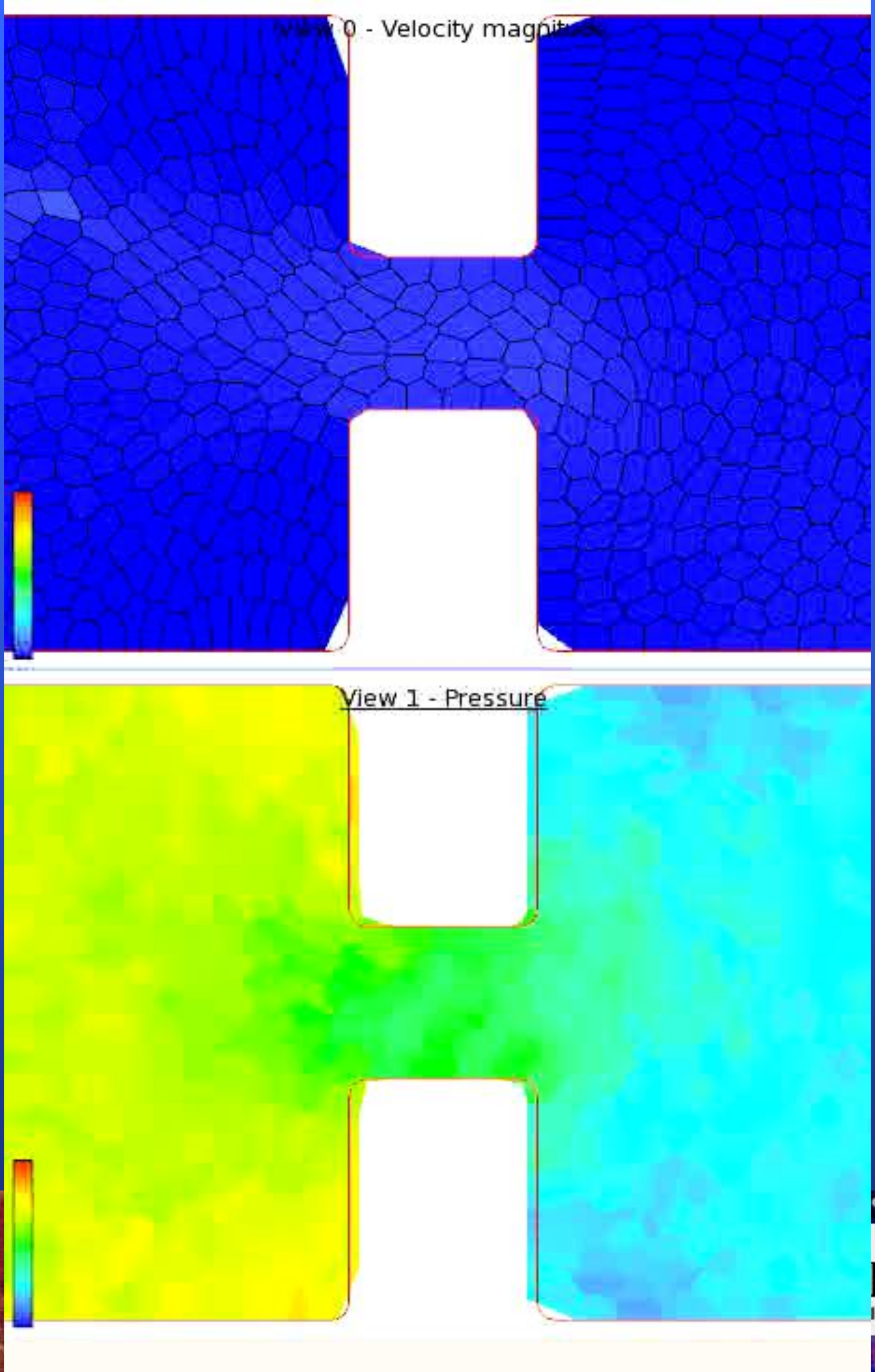
Make small increment in strain and re-converge to equilibrium.

4:1:4 quasi-static constriction flow

Bubbles coloured by
instantaneous velocity (displacement)

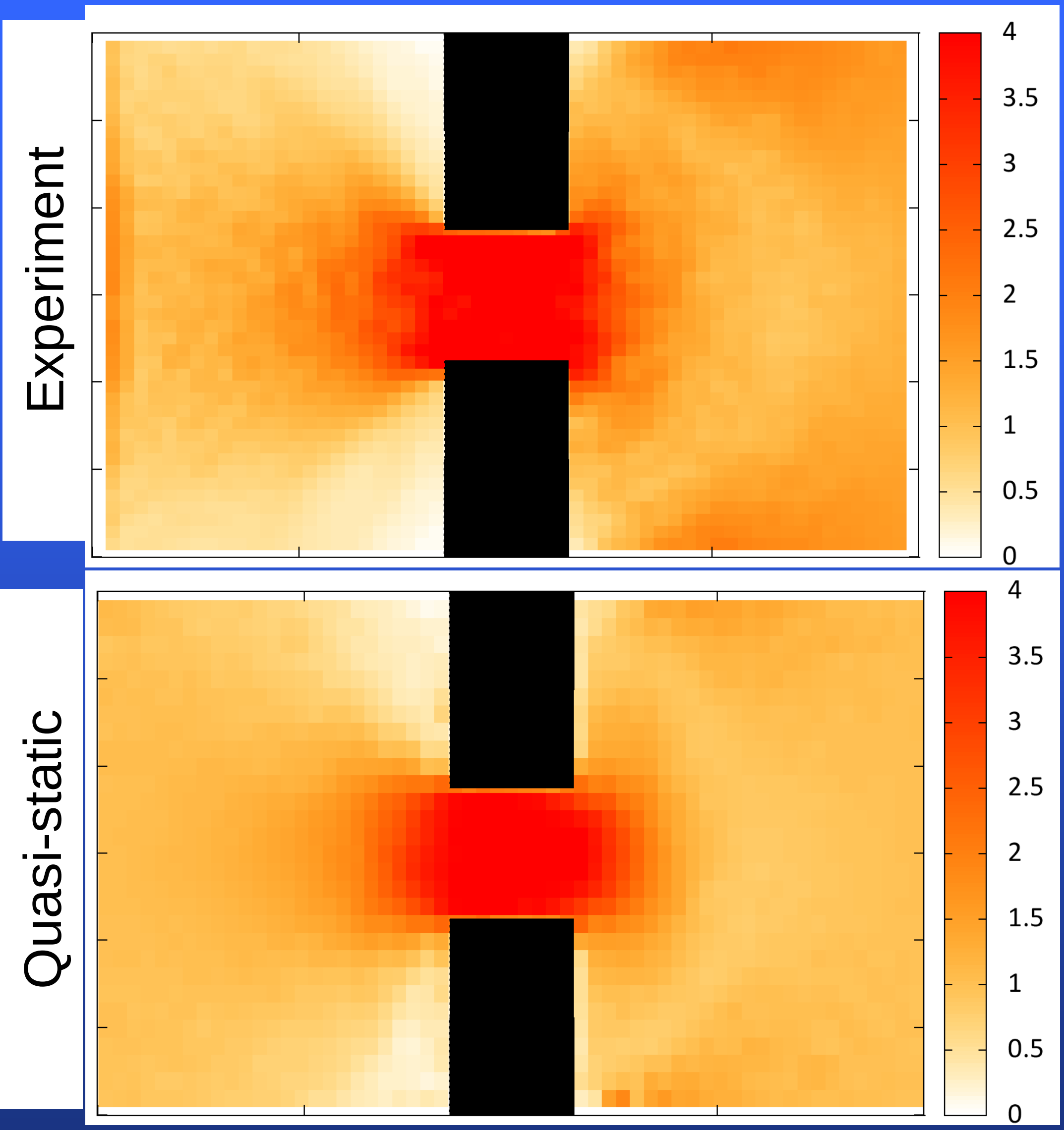
and

pixels coloured
by bubble pressure
averaged over 50 iterations

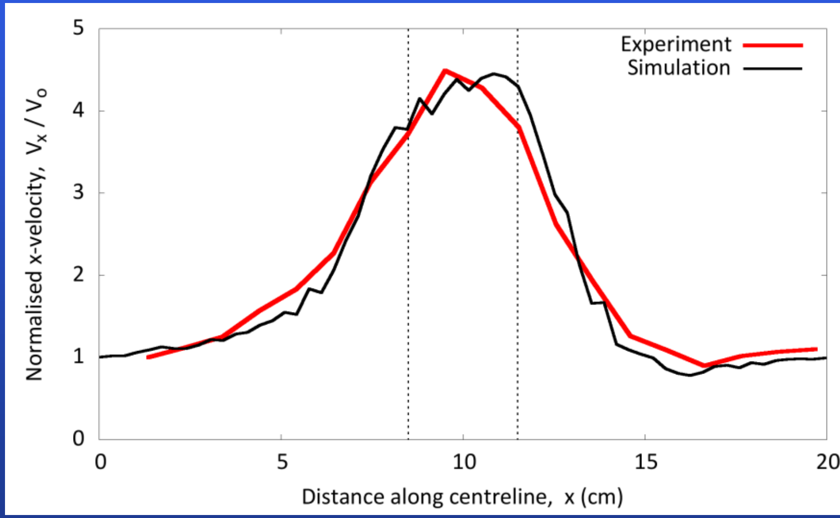


Visualization tool developed by Dan Lipsa (Swansea)

Validation (velocity)



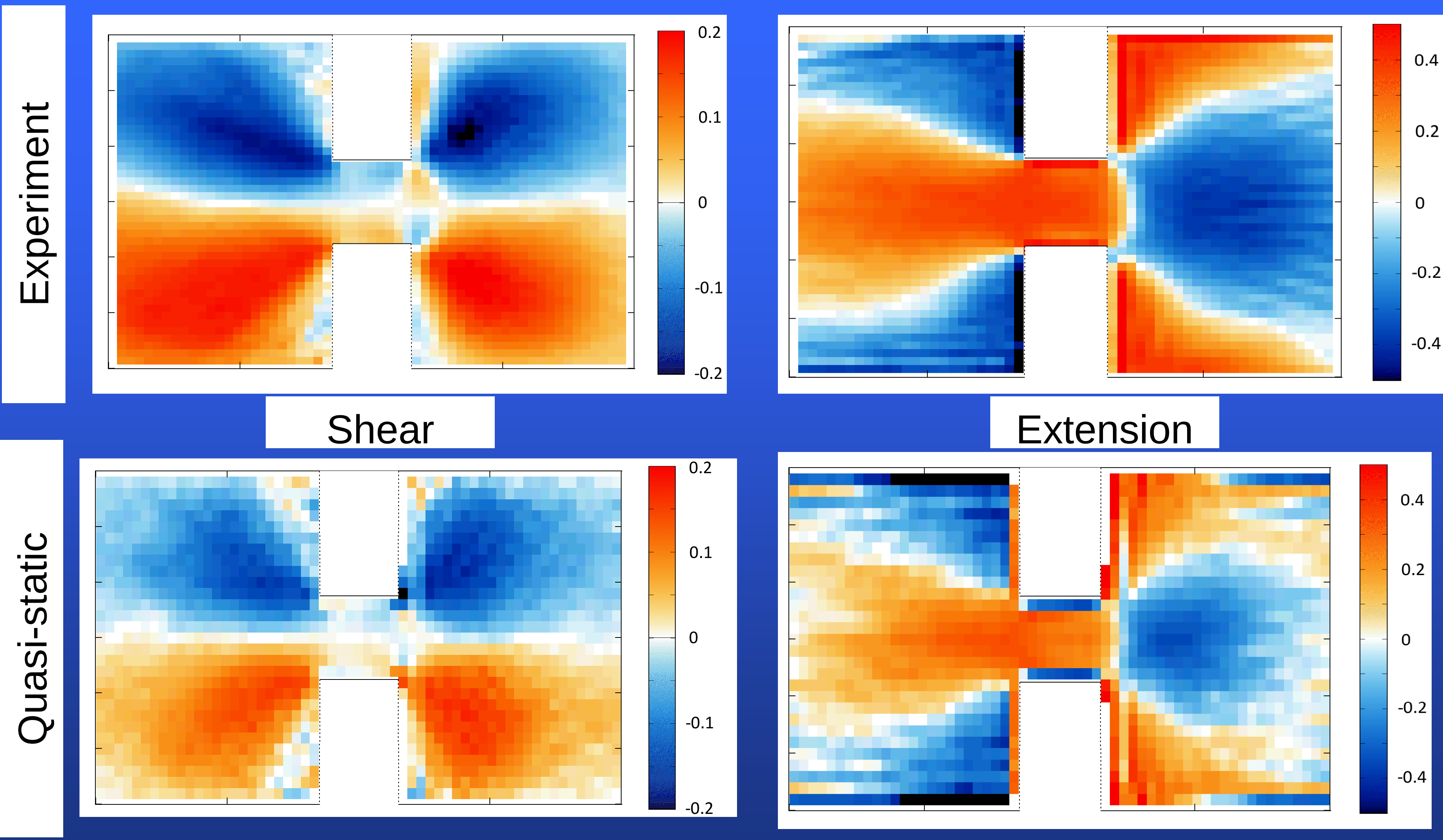
Average velocity magnitude scaled by the free-stream velocity.



Sian Jones, J. Rheol. 2012

Validation (elastic stress)

Sian Jones, J. Rheol. 2012



Foam rheology

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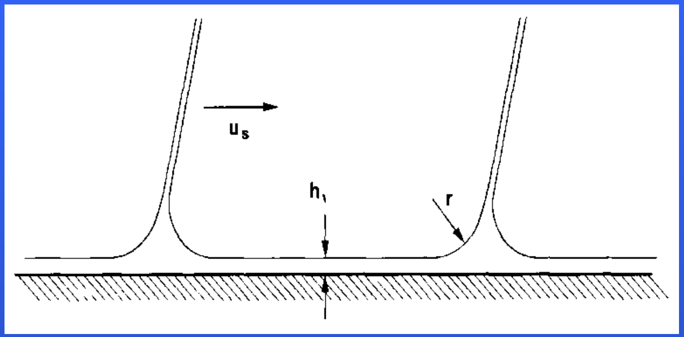


- Structural (mechanical) relaxation

- Friction at boundaries

- Bulk and surface viscosities

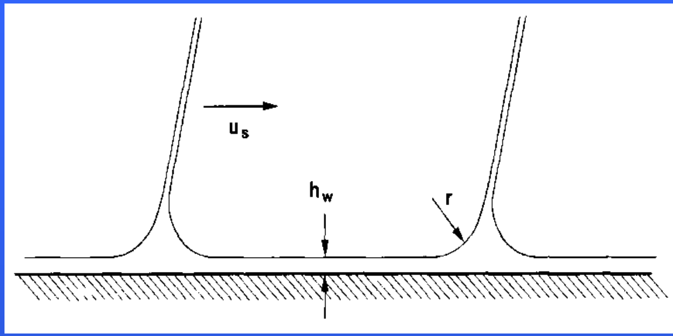
- Equilibration of surfactant concentration
  (within surface and with bulk)

in addition to coarsening (bubble size) and drainage (liquid fraction)

Viscous Froth Model

Kern *et al*. PRE, 2004

Incorporate external dissipation: drag of liquid (Plateau borders) on bounding surfaces. At the same time, retain *both* pressures & curvatures to give a realistic dry structure.



Laplace Law modified to:

with drag λ,
normal velocity v_n,
curvature C,
surface tension γ,
pressure difference Δp.

$$\lambda v_n = \gamma C - \Delta p$$

Limiting cases include:

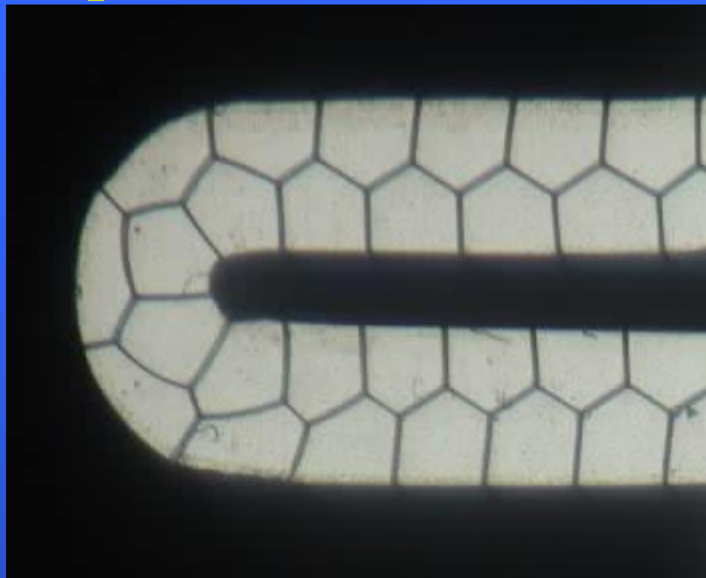
- Ideal soap froth
- Grain growth

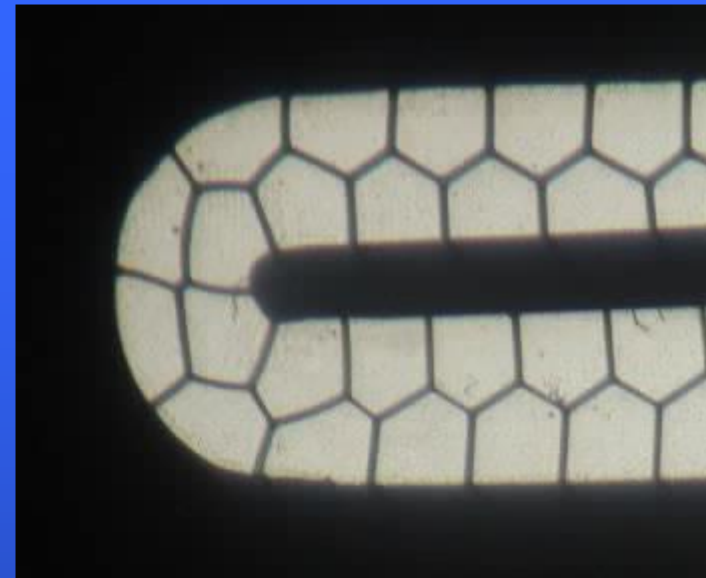
Keep 120º angles, but films are no longer circular arcs

The T_1 generator

Algorithm works: predicts effects of friction, in agreement with experiment.

Experiment

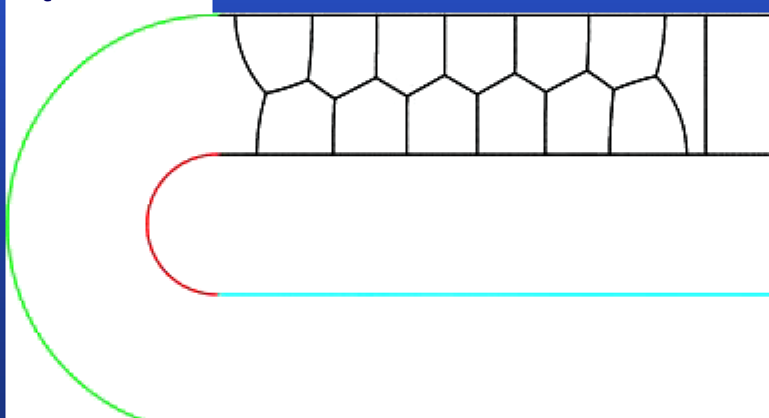




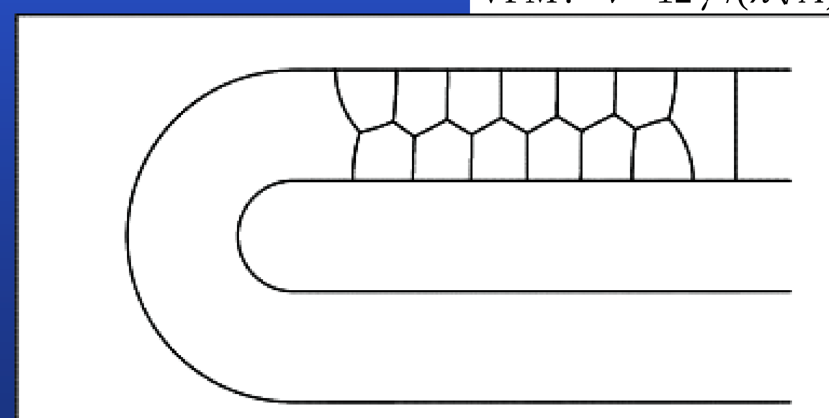
Slow

Fast

Quasistatic

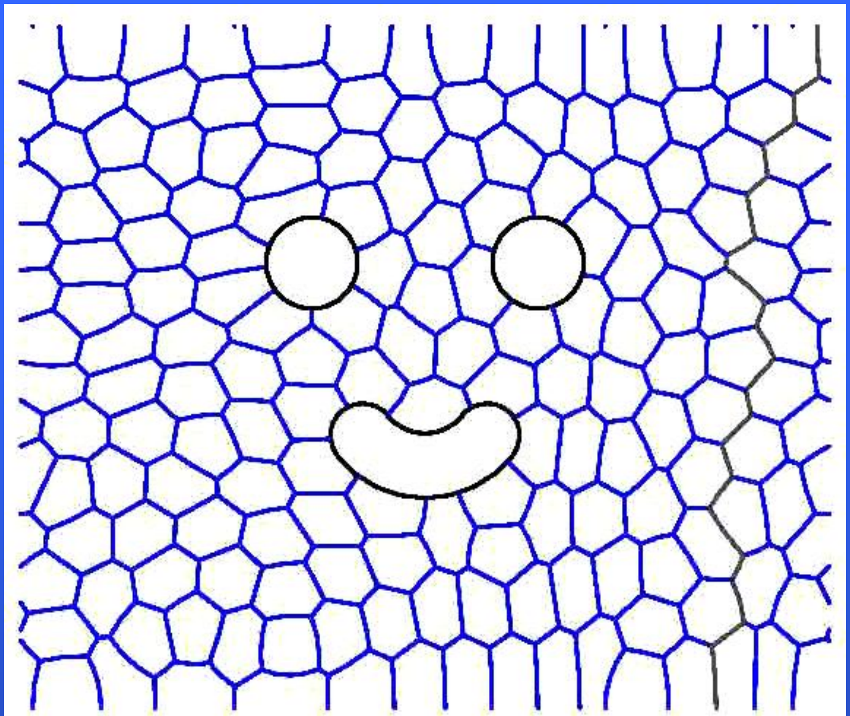


VFM: $v = 12\,\gamma/(\lambda\sqrt{A})$



Drenckhan et al, Coll Surf A, 2005

Summary



The local structure of a foam, determined by area minimization, provides a route to predicting dynamics.

Topological changes are the manifestation of plasticity in a foam and control the forces and stresses within it.

Our quasi-static simulations accurately predict the dynamics of foams in 2D constriction flows at low speeds.

To what extent does the addition of further dissipation mechanisms to the simulations change/improve the predictions?